Chapter 31

Problem 1

The flux is

\[ \Phi_B = BA \]

The rate of change of flux is

\[ \frac{d\Phi_B}{dt} = A \frac{dB}{dt} = \pi (0.11/2)^2 \times 0.16 = 1.5 \times 10^{-3} \text{ V} \]

Problem 4

Part a

\[ \mathcal{E} = - \frac{d\Phi_B}{dt} = - \frac{d}{dt} (BA) = - A \frac{dB}{dt} = - \pi (0.12)^2 \frac{0.5}{2.0} = -1.1 \times 10^{-2} \text{ V} \]

Part b

\[ \mathcal{E} = - A \frac{dB}{dt} = 0 \]

as \( dB/dt = 0 \).

Part c

\[ \mathcal{E} = - A \frac{dB}{dt} = - \pi (0.12)^2 \left( \frac{-0.5}{2.0} \right) = 1.1 \times 10^{-2} \text{ V} \]

Problem 7

Induced emf is

\[ \mathcal{E} = - \frac{d}{dt} (NBA) \]

Here \( A \) is the area of cross-section of the solenoid as the magnetic field is restricted within it. So

\[ \mathcal{E} = - NA \frac{dB}{dt} = - NA \frac{d}{dt} (\mu_0 n_i s) \]
where $n$ is the turn density of the solenoid and $i_s$ is the current in it. So

$$E = -N A \mu_0 n \frac{di_s}{dt}$$

So the current in the coil is

$$i = \frac{E}{R} = -\frac{NA \mu_0 n}{R} \frac{di_s}{dt} = -\frac{120 \times \pi (0.016)^2 \times 4 \pi \times 10^{-7} \times 220 \times 100}{5.3} \times (-1.5 \frac{1}{0.025}) = 3.0 \times 10^{-2} \text{ A}$$

**Problem 11**

**Part a**

The induced emf is:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

But $\mathcal{E} = iR$, and $i = dq/dt$. Hence,

$$R \frac{dq}{dt} = -\frac{d\Phi_B}{dt}$$

This gives:

$$Rdq = -d\Phi_B$$

Integrating this from time zero to $t$, gives:

$$R \int_0^{q(t)} dq = -\int_{\Phi_B(0)}^{\Phi_B(t)} d\Phi_B$$

which is

$$Rq(t) = -[\Phi_B(t) - \Phi_B(0)].$$

Hence,

$$q(t) = \frac{1}{R} [\Phi_B(0) - \Phi_B(t)]$$

**Part b**

$q(t)$ is the net charge flow in time $t$. It could be zero if equal amounts of charge flow in both directions during different periods of the total time $t$.

**Problem 13**

Following problem 11, the initial flux is

$$\Phi_B(0) = NBA$$
and the final flux is
\[ \Phi_B(t) = -NBA \]
So,
\[ q(t) = \frac{1}{R} [\Phi_B(0) - \Phi_B(t)] = \frac{2NBA}{R} = \frac{2 \times 100 \times 1.60 \times 1.20 \times 10^{-3}}{13.0} = 2.95 \times 10^{-2} \text{ C} \]

**Problem 17**

**Part a**

The induced emf is
\[ \mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(N\vec{B} \cdot \vec{A}) = -\frac{d}{dt}(NBA \cos \theta) \]
where \( \theta \) is the angle between \( \vec{B} \) and the area \( \vec{A} \). So
\[ \mathcal{E} = -NBA \frac{d}{dt}(\cos \theta) \]
As frequency is \( f \), \( \theta = \omega t = 2\pi ft \). So
\[ \mathcal{E} = -NBA \frac{d}{dt}(\cos(2\pi ft)) = 2\pi f NBA \sin(2\pi ft) \]
The area \( A = ab \). So
\[ \mathcal{E} = 2\pi f Nab B \sin(2\pi ft) = \mathcal{E}_0 \sin(2\pi ft) \]
where \( \mathcal{E}_0 = 2\pi f Nab B \).

**Part b**

For the specifications given
\[ Nab = \frac{\mathcal{E}_0}{2\pi f B} = 0.796 \text{ m}^2 \]

**Problem 27**
Part a

The induced emf in the closed loop created by the rod and the rails is:

\[ E = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A} = -\frac{d}{dt}(BA) = -B \frac{d}{dt}(Lx) = -BL \frac{dx}{dt} = -BLv = -4.81 \times 10^{-2} \text{ V} \]

Here \( d\mathbf{A} \) is chosen to be directed out of the page and hence positive emf is counterclockwise. The negative result means that the emf is clockwise. As the rails have negligible resistance, this emf is seen completely along the rod.

Part b

The rails have negligible resistance. So the total resistance of the loop is 18.0 \( \Omega \). So the current is:

\[ i = \frac{E}{R} = \frac{-4.81 \times 10^{-2}}{18.0} = -2.67 \times 10^{-3} \text{ A} \]

The negative sign means that the current is clockwise and hence, upwards along the rod.

Part c

The rate of internal energy generation is the resistive power loss:

\[ P = i^2R = (2.67 \times 10^{-3})^2 \times 18.0 = 1.29 \times 10^{-4} \text{ W} \]

Problem 32

Path 1

For the given direction of the loop, the direction of \( d\mathbf{A} \) is out of the page (right-hand-rule). So, from Faraday’s law

\[ \int \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A} = -\frac{d}{dt} \int B dA(-1) = \frac{d}{dt}(BA) = A \frac{dB}{dt} = \pi(0.20)^2 \times (-8.50 \times 10^{-3}) = -1.07 \times 10^{-3} \text{ V} \]

Path 2

For the given direction of the loop, the direction of \( d\mathbf{A} \) is into the page (right-hand-rule). So, from Faraday’s law

\[ \int \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A} = -\frac{d}{dt} \int B dA(-1) = \frac{d}{dt}(BA) = A \frac{dB}{dt} = \pi(0.30)^2 \times (-8.50 \times 10^{-3}) = \]
= -2.40 \times 10^{-3} \text{ V}

Path 3

Once again $d\vec{A}$ is out of the page. So

$$\int \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = -\frac{d}{dt} \left[ \int_{R_1} \vec{B} \cdot d\vec{A} + \int_{R_2} \vec{B} \cdot d\vec{A} \right] = -\frac{d}{dt} \left[ \int_{R_1} B \, dA(-1) + \int_{R_2} B \, dA \right] =$$

$$= -\frac{d}{dt}[-B_1A_1 + B_2A_2] = A_1 \frac{dB_1}{dt} - A_2 \frac{dB_2}{dt} = \pi(0.20)^2 - \pi(0.30)^2 \times (-8.50 \times 10^{-3}) = 1.34 \times 10^{-3} \text{ V}

Problem 40

Part a

$$\mathcal{E}_L = -L \frac{di}{dt}$$

As $\mathcal{E}_L$ and $i$ are in the same direction, $di/dt$ must be negative to make $\mathcal{E}_L$ positive. So $i$ is decreasing.

Part b

$$L = -\frac{\mathcal{E}_L}{di/dt} = -\frac{17}{-25 \times 10^3} = 6.8 \times 10^{-4} \text{ H}

Problem 42

Part a

$$\mathcal{E} = -L \frac{di}{dt} = -4.6 \times \frac{7}{2 \times 10^{-3}} = -16000 \text{ V}

Part b

$$\mathcal{E} = -L \frac{di}{dt} = -4.6 \times \left( -\frac{2}{3 \times 10^{-3}} \right) = 3100 \text{ V}$$
Part c

\[ \mathcal{E} = -L \frac{di}{dt} = -4.6 \times \left( -\frac{5}{1 \times 10^{-3}} \right) = 23000 \text{ V} \]

Problem 44

![Diagram of two inductors in series with currents and voltages labeled]

Part a

From Kirchhoff’s current rule:

\[ i = i_1 + i_2 \iff \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \]

\( \mathcal{E} \) being the same across both inductors:

\[ \mathcal{E} = -L_1 \frac{di_1}{dt}, \quad \mathcal{E} = -L_2 \frac{di_2}{dt}, \quad \mathcal{E} = -L_{eq} \frac{di}{dt} \]

where \( L_{eq} \) is the equivalent inductance of the two inductors. The equations above can be combined to give:

\[ \frac{\mathcal{E}}{L_{eq}} = \frac{\mathcal{E}}{L_1} + \frac{\mathcal{E}}{L_2} \]

Hence,

\[ \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \]

Part b

If the inductors are close, the magnetic field of one may induce an emf in the other.

Part c

For \( N \) inductors in parallel

\[ \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N} = \sum_{i=1}^{N} \frac{1}{L_i} \]
Problem 47

At $t = 0$, $i = i_0 = 1.0$ A. At any other time $t$

$$i = i_0e^{-tR/L}$$

or

$$\ln\left(\frac{i}{i_0}\right) = -tR/L$$

Solving for $R$ gives:

$$R = -\frac{L}{t}\ln\left(\frac{i}{i_0}\right) = \frac{L}{t}\ln\left(\frac{i_0}{i}\right) = \frac{10}{1}\ln\left(\frac{1.0}{10 \times 10^{-3}}\right) = 46 \ \Omega$$

Problem 50

$$\mathcal{E}_L = -L\frac{di}{dt}$$

Hence, from Kirchhoff’s rule:

$$\mathcal{E} = iR + L\frac{di}{dt}$$

$$= R(3.0 + 5.0t) + L\frac{d}{dt}(3.0 + 5.0t)$$

$$= 4.0(3.0 + 5.0t) + 6 \times 5.0$$

$$= (42 + 20t) \text{ V}$$

Problem 53

![Circuit Diagram]

Part a

When the switch is just closed, current in $L$ has still not changed from zero. It will take some time. So $i_3 = 0$. But from Kirchhoff’s current rule: $i_1 = i_2 + i_3$. So, in the present case: $i_1 = i_2$. Hence, from loop 1:

$$\mathcal{E} - i_1R_1 - i_1R_2 = 0.$$
So,
\[ i_1 = \frac{\mathcal{E}}{R_1 + R_2} = \frac{100}{10 + 20} = 3.33 \, \text{A}, \]
and \( i_2 = i_1 \).

**Part b**

A long time later \( i_3 \) becomes a steady current and hence \( di_3/dt = 0 \), and
\[ \mathcal{E}_L = -L \frac{di_3}{dt} = 0. \]
So now the two loop equations are:
\[ \mathcal{E} - i_1 R_1 - i_2 R_2 = 0, \]
\[ i_2 R_2 - i_3 R_3 = 0. \]
But, from Kirchhoff’s current rule: \( i_3 = i_1 - i_2 \). So the two equations can be written as:
\[ \mathcal{E} - i_1 R_1 - i_2 R_2 = 0, \]
\[ i_2 R_2 - (i_1 - i_2) R_3 = 0. \]
Solving these simultaneously on gets:
\[ i_1 = \frac{\mathcal{E}}{R_1 + R_2 R_3/(R_2 + R_3)} = 4.55 \, \text{A}, \]
\[ i_2 = \frac{i_1 R_3}{R_2 + R_3} = 2.73 \, \text{A}, \]
\[ i_3 = i_1 - i_2 = 1.82 \, \text{A}. \]

**Part c**

When the switch is just turned off, \( i_1 = 0 \). But the inductor does not let \( i_3 \) go to zero at once. So, from Kirchhoff’s current rule: \( i_3 = -i_2 \), and \( i_3 \) is still of the value as in part (b). Hence,
\[ i_2 = -i_3 = -1.82 \, \text{A}. \]

**Part d**

As the switch is open \( i_1 = 0 \). After a long time \( i_3 \) becomes steady once again and hence, there is no emf across the inductor. So, loop 2 gives:
\[ i_2 R_2 - i_3 R_3 = 0. \]
But \( i_2 = -i_3 \). So,
\[ i_2(R_2 + R_3) = 0. \]
Hence, \( i_2 = 0 \).
Problem 59

Part a

The current rises as follows.

\[ i = \frac{\mathcal{E}}{R}(1 - e^{-\frac{R t}{L}}) \]

Hence,

\[ e^{-\frac{R t}{L}} = 1 - \frac{iR}{\mathcal{E}} \]

Then,

\[ -\frac{R t}{L} = \ln \left(1 - \frac{iR}{\mathcal{E}}\right) \]

So,

\[ L = \frac{-R t}{\ln(1 - iR/\mathcal{E})} = 97.9 \text{ H} \]

Part b

The stored energy is

\[ U = L i^2 / 2 = 97.9 \times (2.00 \times 10^{-3})^2 / 2 = 1.96 \times 10^{-4} \text{ J} \]