Chapter 30

Problem 1

Part a

The magnetic field is

\[ B = \frac{\mu_0 i}{2\pi r} = \frac{2 \times 10^{-7} \times 100}{6.1} = 3.3 \times 10^{-6} \text{ T} \]

Part b

There will be interference as the wire magnetic field is about 17% of the Earth’s field.

Problem 3

Part a

The magnitude of the current is

\[ i = \frac{2\pi r B}{\mu_0} = \frac{0.08 \times 39 \times 10^{-6}}{2 \times 10^{-7}} = 16 \text{ A} \]

Part b

The direction of the magnetic field must be horizontal and southwards to cancel the Earth’s field.
So the current must be horizontal and eastwards.

Problem 6

\[ a \quad \times \quad b \]
\[ c \quad d \]
Biot Savart law gives:

\[
d\mathbf{B} = \frac{\mu_0 i}{4\pi r^3} d\mathbf{s} \times \mathbf{r}
\]

Along the straight line sections of the wire (labelled a and b), \(d\mathbf{s}\) (as defined) is along the wire and so is \(\mathbf{r}\) (as it must point towards the target which is the center of the circle). This gives: \(d\mathbf{s} \times \mathbf{r} = 0\). Hence, the magnetic field due to these sections is zero.

For the top circular section of the wire (labelled c), \(d\mathbf{s}\) is still along the direction of the current (tangent to the circle). The direction of \(\mathbf{r}\), being towards the center, is in the radial direction. Hence, \(d\mathbf{s}\) and \(\mathbf{r}\) are perpendicular to each other and using the right-hand-rule their cross product is directed into the page. The magnitude of \(d\mathbf{B}\) due to any infinitesimal section \(d\mathbf{s}\) of the wire would then be (current is \(i/2\) as only half the current flows in one semicircular section):

\[
dB = \frac{\mu_0 (i/2)}{4\pi r^3} |d\mathbf{s} \times \mathbf{r}| = \frac{\mu_0 i}{8\pi r^2} ds
\]

as the angle between \(d\mathbf{s}\) and \(\mathbf{r}\) is 90 degrees and the magnitude of the vector \(\mathbf{r}\) is \(r\). Integrating this over the semicircular section (note that \(r\) is a constant) one gets:

\[
B = \frac{\mu_0 i}{8\pi r^2} \int_0^{\pi r} ds = \frac{\mu_0 i}{8\pi r^2} \pi r = \frac{\mu_0 i}{8r}
\]

The magnetic field due to the bottom semicircular section is found to be the same in magnitude but opposite in direction as the current is oppositely directed (counterclockwise). Hence, the total magnetic field due to all sections of the wire is zero.

**Problem 13**

Magnetic field due to the piece \(d\mathbf{s}\) is:

\[
d\mathbf{B} = \frac{\mu_0 i}{4\pi r^3} d\mathbf{s} \times \mathbf{r}
\]

So the direction is out of the page and the magnitude

\[
dB = \frac{\mu_0 i}{4\pi r^2} |d\mathbf{s} \times \mathbf{r}| = \frac{\mu_0 i}{4\pi r^2} ds \sin \theta
\]

If we choose the wire to be along the \(x\) axis with the origin at its right end,

\[
 ds = dx, \quad r = (x^2 + R^2)^{1/2}, \quad \sin \theta = \frac{R}{r} = \frac{R}{(x^2 + R^2)^{1/2}}
\]
Hence,
\[ dB = \frac{\mu_0 i R}{4\pi} \frac{dx}{(x^2 + R^2)^{3/2}} \]

Integrating this along the length of the wire (limits from \(-L\) to 0) gives:
\[
B = \int dB = \frac{\mu_0 i R}{4\pi} \int_{-L}^{0} \frac{dx}{(x^2 + R^2)^{3/2}} = \frac{\mu_0 i R}{4\pi} \left[ \frac{x}{R^2(x^2 + R^2)^{1/2}} \right]_{-L}^{0} = \frac{\mu_0 i L}{4\pi R(L^2 + R^2)^{1/2}}
\]

**Problem 23**

The current in wire 2 must be out of the page to produce an opposite field to cancel the field of wire 1. As the total magnetic field must be zero at \(P\)

\[
\frac{\mu_0 i_1}{2\pi r_1} = \frac{\mu_0 i_2}{2\pi r_2}
\]

Hence,
\[
\frac{6.5}{2.25} = \frac{i_2}{1.5}
\]

and
\[
i_2 = \frac{6.5 \times 1.5}{2.25} = 4.3 \text{ A}
\]

**Problem 24**

**Wire 1 (from left)**

Magnetic field due to the other four wires is in the \(\hat{\mathbf{k}}\) direction. So
\[
\mathbf{B} = -\left( \frac{\mu_0 i}{2\pi d} + \frac{\mu_0 i}{4\pi d} + \frac{\mu_0 i}{6\pi d} + \frac{\mu_0 i}{8\pi d} \right) \mathbf{k} = \frac{25\mu_0 i}{24\pi d} \mathbf{k}
\]

The force on the wire is
\[
\mathbf{F} = i\mathbf{L} \times \mathbf{B} = i(L\mathbf{\hat{i}}) \times \left( -\frac{25\mu_0 i}{24\pi d} \mathbf{k} \right) = \frac{25\mu_0 i^2 L}{24\pi d} \mathbf{j}
\]

For \(L = 1 \text{ m}\)
\[
\mathbf{F} = \frac{25\mu_0 i^2}{24\pi d} \mathbf{j} = 4.69 \times 10^{-5} \mathbf{j} \text{ N}
\]
Wire 2

The magnetic field due to the other four wires is

\[
\mathbf{B} = \frac{\mu_0 i}{2\pi d} \mathbf{k} - \frac{\mu_0 i}{2\pi d} \mathbf{\hat{k}} - \frac{\mu_0 i}{4\pi d} \mathbf{k} - \frac{\mu_0 i}{4\pi d} \mathbf{k} = \frac{-5\mu_0 i}{12\pi d} \mathbf{k}
\]

Again, for a one meter length \(\mathbf{L} = \mathbf{i}\). So the force is

\[
\mathbf{F} = i\mathbf{L} \times \mathbf{B} = i\mathbf{i} \times \left( -\frac{5\mu_0 i}{12\pi d} \mathbf{k} \right) = \frac{5\mu_0 i^2}{12\pi d} \mathbf{j} = 1.88 \times 10^{-5} \mathbf{j} \text{ N}
\]

Wire 3

The magnetic field due to the other four wires is

\[
\mathbf{\hat{B}} = \frac{\mu_0 i}{4\pi d} \mathbf{\hat{k}} + \frac{\mu_0 i}{2\pi d} \mathbf{\hat{k}} - \frac{\mu_0 i}{2\pi d} \mathbf{\hat{k}} - \frac{\mu_0 i}{4\pi d} \mathbf{\hat{k}} = 0
\]

Hence, the force is

\[
\mathbf{\hat{F}} = 0
\]

Wire 4

The magnetic field due to the other four wires is

\[
\mathbf{\hat{B}} = \frac{\mu_0 i}{6\pi d} \mathbf{\hat{k}} + \frac{\mu_0 i}{4\pi d} \mathbf{\hat{\mathbf{k}}} + \frac{\mu_0 i}{2\pi d} \mathbf{\hat{k}} - \frac{\mu_0 i}{2\pi d} \mathbf{\hat{k}} = \frac{5\mu_0 i}{12\pi d} \mathbf{k}
\]

Hence, the force is

\[
\mathbf{\hat{F}} = i\mathbf{L} \times \mathbf{\hat{B}} = i\mathbf{i} \times \left( \frac{5\mu_0 i}{12\pi d} \mathbf{k} \right) = \frac{5\mu_0 i^2}{12\pi d} \mathbf{j} = -1.88 \times 10^{-5} \mathbf{j} \text{ N}
\]

Wire 5

The magnetic field due to the other four wires is

\[
\mathbf{\hat{B}} = \frac{\mu_0 i}{8\pi d} \mathbf{\hat{k}} + \frac{\mu_0 i}{6\pi d} \mathbf{\hat{k}} + \frac{\mu_0 i}{4\pi d} \mathbf{\hat{\mathbf{k}}} + \frac{\mu_0 i}{2\pi d} \mathbf{\hat{\mathbf{k}}} = \frac{25\mu_0 i}{24\pi d} \mathbf{k}
\]

Hence, the force is

\[
\mathbf{\hat{F}} = i\mathbf{L} \times \mathbf{\hat{B}} = i\mathbf{i} \times \left( \frac{25\mu_0 i}{24\pi d} \mathbf{k} \right) = \frac{25\mu_0 i^2}{24\pi d} \mathbf{j} = -4.69 \times 10^{-5} \mathbf{j} \text{ N}
\]
Problem 29

Magnetic field due to the long straight wire near the rectangle is into the page. The force on an element of wire on the left branch of the rectangle is exactly opposed by the force on an element of wire on the right branch if the two elements are at the same distance from the long wire. Hence, the forces on the left and right branches of the rectangle add to zero. On the top branch the force is
\[ \vec{F}_1 = i_r \vec{L} \times \vec{B}_1 \]
where \( i_r \) is the current in the rectangle. So the direction of \( \vec{F}_1 \) is upwards and the magnitude is
\[ F_1 = i_r |\vec{L} \times \vec{B}_1| = i_r LB_1 = i_r L \frac{\mu_0 i_s}{2\pi a} \]
where \( i_s \) is the current in the straight wire. As the current in the bottom branch of the rectangle is in the opposite direction, the force on it is downwards and its magnitude is
\[ F_2 = i_r LB_2 = i_r L \frac{\mu_0 i_s}{2\pi (a + b)} \]
So the net upward force is
\[ F = F_1 - F_2 = \frac{i_r L \mu_0 i_s}{2\pi} \left( \frac{1}{a} - \frac{1}{a + b} \right) = 3.2 \times 10^{-3} \, \text{N} \]

Problem 31

Part a

As the direction of integration is clockwise, positive currents are into the page. So, using Ampere’s law
\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 (2 - 2 - 2) = -8\pi \times 10^{-7} \, \text{Tm} \]

Part b

Similarly, in this case
\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 (2 + 2 - 2 - 2) = 0 \]
Problem 35

Choose a loop of integration that is a circle of radius \( r \) \((r < a)\). Then

\[
\oint \mathbf{B} \cdot d\mathbf{s} = \oint B \, ds = B \oint ds = B(2\pi r)
\]
as \( B \) is a constant on the loop due to cylindrical symmetry. Current within the loop is

\[
i = \int \mathbf{J} \cdot d\mathbf{A} = \int J \, dA
\]

For an infinitesimal ring of radius \( r' \), the area is

\[
dA = 2\pi r' \, dr'
\]

Hence, the current inside the loop of radius \( r \) is

\[
i = \int_0^r \frac{J_0 r'}{a} 2\pi r' \, dr' = \frac{2\pi J_0}{a} \int_0^r r'^2 \, dr' = \frac{2\pi J_0 \, r^3}{3a}
\]

So, from Ampere’s law

\[
B(2\pi r) = \mu_0 \frac{2\pi J_0 \, r^3}{3a}
\]

This gives

\[
B = \frac{\mu_0 J_0 \pi r^2}{3a}
\]

Problem 41

\[
B = \mu_0 ni = 4\pi \times 10^{-7} \times \frac{200}{0.25 \times 0.30} = 3.0 \times 10^{-4} \text{ T}
\]