Chapter 25

Problem 2

Potential energy difference is

$$\Delta U = q\Delta V = 1.60 \times 10^{-19} \times 1.2 \times 10^9 \ J = \frac{1.60 \times 10^{-19} \times 1.2 \times 10^9}{1.60 \times 10^{-19}} \ eV = 1.2 \times 10^9 \ eV$$

Problem 5

The potential difference is

$$\Delta V = -\int \vec{E} \cdot d\vec{s} = - \int \frac{\sigma}{2\epsilon_0} ds(-1) = \frac{\sigma}{2\epsilon_0} s$$

If the separation between equipotentials is $s$, then $\Delta V = 50 \ V$ and

$$s = \frac{2\Delta V \epsilon_0}{\sigma} = 8.9 \times 10^{-3} \ m$$

Problem 6

Part a

The electric field is

$$E = \frac{F}{q} = \frac{3.9 \times 10^{-15}}{1.60 \times 10^{-19}} = 2.4 \times 10^4 \ N/C$$

Part b

The potential difference is

$$\Delta V = Ed = 2.9 \times 10^3 \ V$$
Problem 8

Part a

The potential difference between any two points ‘a’ and ‘b’ is defined to be:

\[ V_b - V_a = - \int_a^b \vec{E} \cdot d\vec{s} \]

If the reference is at ‘a’ which is at \( r = 0 \) and if ‘b’ is at any arbitrary \( r \), then \( V(r) = V_b \) and \( V_a = 0 \). Hence, if the integration path is chosen along a radial line:

\[ V(r) = - \int_0^r \vec{E} \cdot d\vec{s} = - \int_0^r E \, dr = - \int_0^r \frac{qr}{4\pi \varepsilon_0 R^3} \, dr = -\frac{qr^2}{8\pi \varepsilon_0 R^3} \]

Part b

Using the above formula for \( V(r) \),

\[ V(R) = -\frac{qR^2}{8\pi \varepsilon_0 R^3} = -\frac{q}{8\pi \varepsilon_0 R} \]

Part c

As the above result is negative, the center is at the higher potential.

Problem 13

Part a

\[ V_A - V_B = \frac{q}{4\pi \varepsilon_0 d_1} - \frac{q}{4\pi \varepsilon_0 d_2} = -4.5 \times 10^3 \, V \]

Part b

As the distances are the same the answer is the same as in part a.
Problem 15

Potential is zero a distance $x$ from the origin if (for positive $x$)

$$\frac{q_1}{4\pi\varepsilon_0 x} + \frac{q_2}{4\pi\varepsilon_0 (d - x)} = 0$$

Replacing $q_1 = q$ and $q_2 = -3q$ gives

$$\frac{q}{4\pi\varepsilon_0 x} - \frac{3q}{4\pi\varepsilon_0 (d - x)} = 0$$

or

$$\frac{1}{x} = \frac{3}{d - x}$$

Solving this for $x$ gives

$$x = d/4$$

For negative $x$ values

$$-\frac{q_1}{4\pi\varepsilon_0 x} + \frac{q_2}{4\pi\varepsilon_0 (d - x)} = 0$$

Then

$$-\frac{q}{4\pi\varepsilon_0 x} - \frac{3q}{4\pi\varepsilon_0 (d - x)} = 0$$

Solving for $x$ gives

$$x = -d/2$$

Problem 25

Part a

Potential due to an element of charge $dq$ is

$$dV = \frac{dq}{4\pi\varepsilon_0 r}$$

If the point of interest is the center of the circle then $r = R$. So

$$dV = \frac{dq}{4\pi\varepsilon_0 R}$$

As $R$ is a constant for all points on the circle, integrating $dV$ gives

$$V = \frac{1}{4\pi\varepsilon_0 R} \int dq$$

Integrating $dq$ over the complete circle gives the total charge

$$\int dq = Q - 6Q = -5Q$$

So

$$V = \frac{-5Q}{4\pi\varepsilon_0 R}$$
Part b

For point $P$

$$r = (R^2 + z^2)^{1/2}$$

So

$$dV = \frac{dq}{4\pi\varepsilon_0 \sqrt{R^2 + z^2}}$$

As $R$ and $z$ are both constants for all points on the circle, integrating $dV$ gives

$$V = \frac{1}{4\pi\varepsilon_0 \sqrt{R^2 + z^2}} \int dq = \frac{-5Q}{4\pi\varepsilon_0 \sqrt{R^2 + z^2}}$$

Problem 29

The charge on the element of length $dx$ is $dq = \lambda dx$. So the potential due to it is

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{\lambda dx}{4\pi\varepsilon_0 (x + d)} = \frac{cx dx}{4\pi\varepsilon_0 (x + d)}$$

Integrating this over the full length gives

$$V = \int_0^L \frac{cx dx}{4\pi\varepsilon_0 (x + d)} = \frac{c}{4\pi\varepsilon_0} \int_0^L \frac{x dx}{x + d} = \frac{c}{4\pi\varepsilon_0} \left[ L - d \ln \left( \frac{L + d}{d} \right) \right]$$

Problem 33

Part a

The potential at a distance $z$ along the axis due to a charge element $dq$ is

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}$$
But it is seen that \( r = \sqrt{z^2 + R^2} \) and it is a constant for all points on the circle. Hence,

\[
V = \frac{1}{4\pi\varepsilon_0 \sqrt{z^2 + R^2}} \int_0^q dq = \frac{q}{4\pi\varepsilon_0 \sqrt{z^2 + R^2}}
\]

**Part b**

So the \( z \) component of the electric field is:

\[
E_z = -\frac{\partial V}{\partial z} = -\frac{q}{4\pi\varepsilon_0} (-1/2)(R^2 + z^2)^{-3/2}2z = \frac{qz}{4\pi\varepsilon_0 (R^2 + z^2)^{3/2}}
\]