**Chapter 23**

**Problem 5**

\[ E = \frac{k|q|}{r^2} \]

Hence,

\[ |q| = \frac{E r^2}{k} = \frac{2.0 \times (0.50)^2}{8.99 \times 10^9} = 5.6 \times 10^{-11} \text{ C} \]

**Problem 13**

Distance of each charge from the center is \( r = \sqrt{a^2 + a^2}/2 = a/\sqrt{2} \). So, adding the electric fields due to the four charges

\[
\vec{E} = \frac{kq}{r^2} (\cos 45^\circ \hat{i} - \sin 45^\circ \hat{j}) + \frac{k(2q)}{r^2} (\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j}) + \frac{k(2q)}{r^2} (-\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j}) + \frac{kq}{r^2} (-\cos 45^\circ \hat{i} - \sin 45^\circ \hat{j})
\]

\[
= \frac{2kq}{r^2} \sin 45^\circ \hat{j} = \frac{4kq}{a^2} \sin 45^\circ \hat{j} = 1.0 \times 10^5 \text{ N/C}
\]

**Problem 16**

The electric field due to the positive charge is (\( R \) and \( \theta \) shown in figure)

\[ \vec{E}_1 = \frac{kq}{R^2} (\cos \theta \hat{i} - \sin \theta \hat{j}) \]

The electric field due to the negative charge is

\[ \vec{E}_2 = \frac{kq}{R^2} (-\cos \theta \hat{i} - \sin \theta \hat{j}) \]
So the total electric field is
\[ \vec{E} = \vec{E}_1 + \vec{E}_2 = -\frac{2kq}{R^2} \sin \theta \hat{j} \]

As \( \sin \theta = d/(2R) \), this gives
\[ \vec{E} = -\frac{2kq d}{2R^2} \hat{j} = -\frac{kqd}{R^3} \hat{j} \]

Using the Pythagorean theorem \( R = (r^2 + (d/2)^2)^{1/2} \). Hence,
\[ \vec{E} = -\frac{kqd}{(r^2 + (d/2)^2)^{3/2}} \hat{j} \]

If \( r \) is much larger than \( d \) \( (r \gg d) \), then
\[ \vec{E} = -\frac{kqd}{r^3} \hat{j} = -\frac{kq}{r^3} \hat{j} = -\frac{k\vec{p}}{r^3} \]

The last step uses the fact that the direction of the dipole moment is along the \( \hat{j} \) direction. Hence \( \vec{p} = p \hat{j} \).

Problem 21

For the top quarter circle, the charge density is \( \lambda = 2q/(\pi r) \). The electric field due to the element \( ds \) is \( d\vec{E}_1 \). The \( x \) and \( y \) components of this field are:
\[ dE_{1x} = dE_1 \cos \theta = \frac{k\lambda ds}{r^2} \cos \theta \]
\[ dE_{1y} = dE_1 \sin \theta = \frac{k\lambda ds}{r^2} \sin \theta \]

From the definition of an angle in radians \( ds = r d\theta \). Hence,
\[ dE_{1x} = \frac{k\lambda}{r} \cos \theta d\theta, \quad dE_{1y} = \frac{k\lambda}{r} \sin \theta d\theta \]

Integration leads to:
\[ E_{1x} = \int dE_{1x} = \frac{k\lambda}{r} \int_{-\pi/2}^{0} \cos \theta d\theta = \frac{k\lambda}{r} \]

\[ E_{1y} = \int dE_{1y} = \frac{k\lambda}{r} \int_{-\pi/2}^{0} \sin \theta d\theta = 0 \]
\[ E_{1y} = \int dE_{1y} = \frac{k\lambda}{r} \int_{-\pi/2}^{0} \sin \theta \, d\theta = -\frac{k\lambda}{r} \]

The computation of electric field due to the bottom quarter circle is similar to that of the top. The differences are the charge density, which is \( -\lambda \), and the limits of integration, which are \( 0 \) and \( \pi/2 \). This gives:

\[ E_{2x} = \int dE_{2x} = \frac{k(-\lambda)}{r} \int_{0}^{\pi/2} \cos \theta \, d\theta = -\frac{k\lambda}{r} \]
\[ E_{2y} = \int dE_{2y} = \frac{k(-\lambda)}{r} \int_{0}^{\pi/2} \sin \theta \, d\theta = -\frac{k\lambda}{r} \]

Now the total electric field due to both quarter circles can be computed:

\[ E_x = E_{1x} + E_{2x} = \frac{k\lambda}{r} - \frac{k\lambda}{r} = 0 \]
\[ E_y = E_{1y} + E_{2y} = -\frac{k\lambda}{r} - \frac{k\lambda}{r} = -\frac{2k\lambda}{r} = -\frac{2k}{r} \frac{2q}{\pi r^2} = -\frac{4kq}{\pi r^2} \]

**Problem 24**

From the symmetry of the problem it can be seen that the left half of the wire produces an \( x \) component of electric field that cancels exactly the \( x \) component produced by the right half of the wire. Hence the total \( x \) component of the electric field \( E_x = 0 \). For the \( y \) component we see that an element of wire of length \( dx \) produces the field

\[ dE_y = dE \sin \theta = \frac{k\lambda}{r^2} dx \sin \theta = \frac{k\lambda}{r^2} \frac{dy}{r} = \frac{k\lambda y}{r^2} \frac{dx}{(y^2 + x^2)^{3/2}} \]

Integrating this from \( -L/2 \) to \( L/2 \) gives:

\[ E_y = \int dE_y = k\lambda y \int_{-L/2}^{L/2} \frac{dx}{(y^2 + x^2)^{3/2}} = \frac{k\lambda y}{y^2} \left[ \frac{x}{\sqrt{y^2 + x^2}} \right]_{x=-L/2}^{x=L/2} = \frac{2k\lambda}{y} \frac{L}{\sqrt{y^2 + L^2}} \]

As \( \lambda = q/L \), this gives:

\[ E_y = \frac{2kq}{y\sqrt{4y^2 + L^2}} \]
Problem 28

The force on a charge \( q \) due to an electric field \( \vec{E} \) is

\[
\vec{F} = q \vec{E}
\]

From Newton’s second law of motion \( \vec{F} = m \vec{a} \). Hence,

\[
m \vec{a} = q \vec{E}
\]

If east is the \( x \)-direction then

\[
\vec{E} = \frac{m \vec{a}}{q} = \frac{9.11 \times 10^{-31}}{(-1.60 \times 10^{-19})} \times 1.80 \times 10^9 \vec{i} = -1.02 \times 10^{-2} \vec{i} \text{ N/C}
\]

Problem 33

Part a

The magnitude of the electric field is

\[
E = \frac{F}{|q|} = \frac{3.0 \times 10^{-6}}{2.0 \times 10^{-9}} = 1.5 \times 10^3 \text{ N/C}
\]

Part b

Magnitude of force on a proton is

\[
F = |q| E = 1.60 \times 10^{-19} \times 1.5 \times 10^3 = 2.4 \times 10^{-16} \text{ N}
\]

The direction is upwards.

Part c

Gravitational force on a proton is

\[
F_g = mg = 1.67 \times 10^{-27} \times 9.8 = 1.64 \times 10^{-26} \text{ N}
\]

Part d

The ratio of the two forces is

\[
\frac{F}{F_g} = 1.5 \times 10^{10}
\]
Problem 35

Part a

Acceleration of the proton is

\[ a = \frac{F}{m} = \frac{qE}{m} = \frac{1.60 \times 10^{-19} \times 2.00 \times 10^4}{1.67 \times 10^{-27}} = 1.92 \times 10^{12} \text{ m/s}^2 \]

Part b

If the proton starts at a speed of zero and ends up with a speed \( v \) then, using one of the equations for constant acceleration we get

\[ v^2 = 2ax \]

where \( x \) is the displacement. Then

\[ v = \sqrt{2ax} = \sqrt{2 \times 1.92 \times 10^{12} \times 0.010} = 1.96 \times 10^5 \text{ m/s} \]

Problem 43

The initial velocity components are:

\[ v_{0x} = v_0 \cos \theta = 6.00 \times 10^6 \times \cos 45^\circ = 4.24 \times 10^6 \text{ m/s} \]
\[ v_{0y} = v_0 \sin \theta = 6.00 \times 10^6 \times \sin 45^\circ = 4.24 \times 10^6 \text{ m/s} \]

The acceleration is due to an electric force and it is in the downward (negative \( y \)) direction. Its value is:

\[ a_y = \frac{F_y}{m} = \frac{qE_y}{m} = \frac{-1.60 \times 10^{-19} \times 2000}{9.11 \times 10^{-31}} = -3.51 \times 10^{14} \text{ m/s}^2 \]

At any point of the trajectory, the \( y \) displacement and \( y \) component of velocity \( (v_y) \) are related as follows (constant acceleration mechanics):

\[ 2a_y y = v_y^2 - v_{0y}^2 \]
To check if the particle hits the top plate, we find the maximum height of the parabolic trajectory and see if it is greater than the distance between the plates. At this maximum height \( y = y_m \), \( v_y \) must be zero. Hence, from the above equation:

\[
y_m = -\frac{v_0^2}{2a_y} = 0.0256 \text{ m}
\]

As this is greater than \( d \), the distance between the plates, the particle will hit the upper plate before it reaches its maximum height. The distance it moves in the \( x \) direction before striking the plate is given by (as there is no acceleration in the \( x \) direction):

\[
x = v_0 x t
\]

where \( t \) is the time at which the particle strikes the upper plate. To find \( t \), we use the following (constant acceleration mechanics):

\[
y = v_0 y t + a_y t^2 / 2
\]

At the time of striking \( y = d \). So, \( t \) is given by:

\[
d = v_0 y t + a_y t^2 / 2
\]

Solving this quadratic equation for \( t \) gives the following two solutions.

\[
t = \frac{-2v_0 y \pm \sqrt{4v_0^2 y + 8a_y d}}{2a_y}
\]

We pick the smaller of the above two solutions as the larger value is the later time at which the parabola would have reached \( y = d \) again if it had not already stopped. So,

\[
t = \frac{-2v_0 y + \sqrt{4v_0^2 y + 8a_y d}}{2a_y} = 6.43 \times 10^{-9} \text{ s}
\]

Hence,

\[
x = v_0 x t = 0.0273 \text{ m}
\]