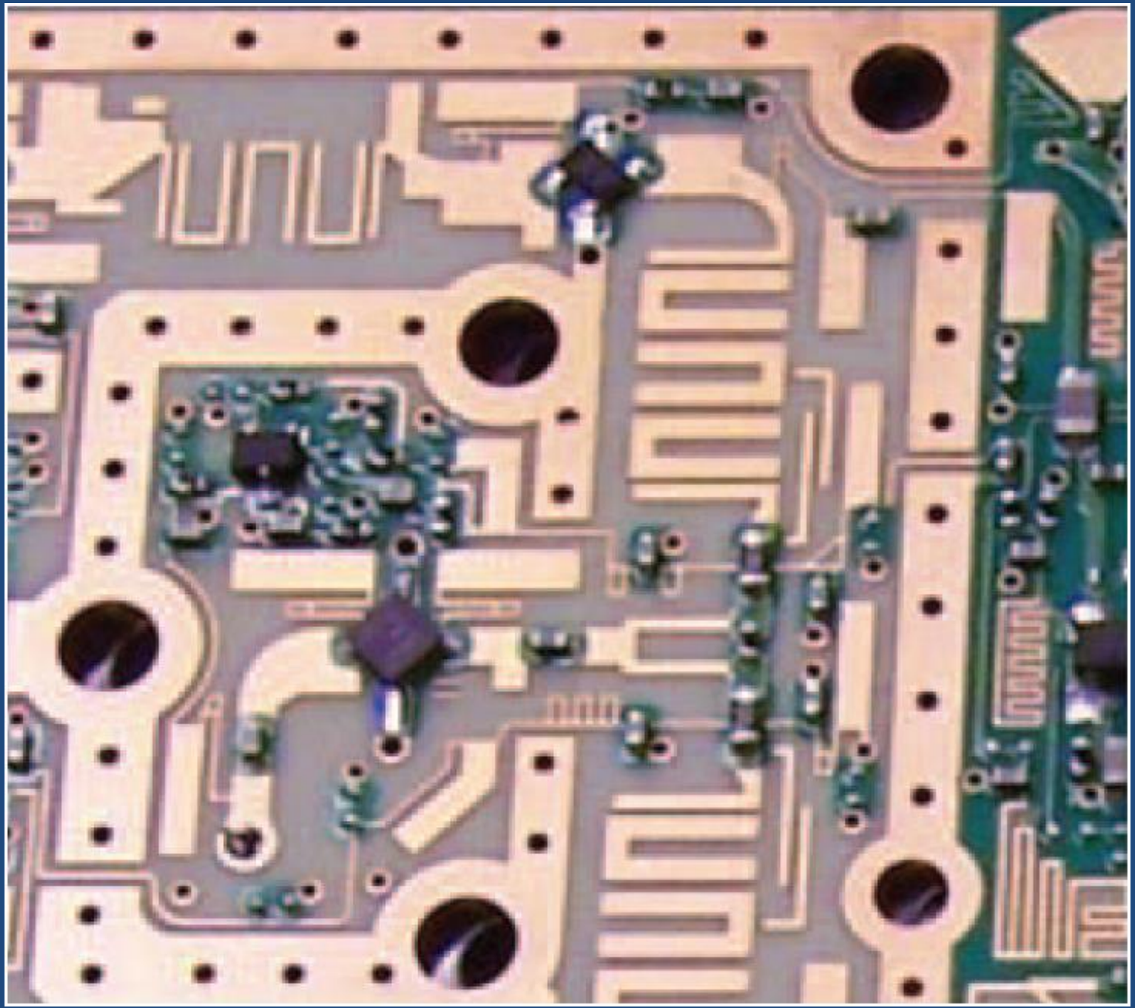


2. TRANSMISSION LINES

7e Applied EM by Ulaby and Ravaioli

Chapter 2 Overview

Chapter Contents

Objectives

Upon learning the material presented in this chapter, you should be able to:

1. Calculate the line parameters, characteristic impedance, and propagation constant of coaxial, two-wire, parallel-plate, and microstrip transmission lines.
2. Determine the reflection coefficient at the load-end of the transmission line, the standing-wave pattern, and the locations of voltage and current maxima and minima.
3. Calculate the amount of power transferred from the generator to the load through the transmission line.
4. Use the Smith chart to perform transmission-line calculations.
5. Analyze the response of a transmission line to a voltage pulse.

Transmission Lines

A transmission line connects a generator to a load

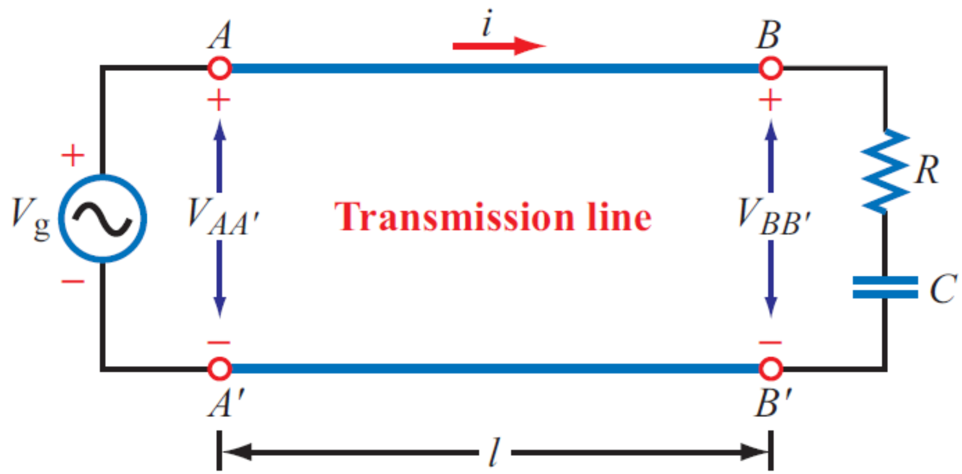
Figure 2-1 A transmission line is a two-port network connecting a generator circuit at the sending end to a load at the receiving end.

Transmission lines include:

- Two parallel wires
- Coaxial cable
- Microstrip line
- Optical fiber
- Waveguide
- etc.

Transmission Line Effects

Is the pair of wires connecting the voltage source to the RC load a transmission line?
Yes.
The wires were ignored in circuits courses.
Can we always ignore them? Not always.



$$V_{AA'} = V_g(t) = V_0 \cos \omega t \qquad \text{(V)}$$

$$\begin{aligned} V_{BB'}(t) &= V_{AA'}(t - l/c) \qquad \text{Delayed by } l/c \\ &= V_0 \cos[\omega(t - l/c)] \\ &= V_0 \cos(\omega t - \phi_0), \end{aligned}$$

At $t = 0$, and for $f = 1$ kHz , if:

(1) $l = 5$ cm:

$$V_{BB'} = V_0 \cos(2\pi f l/c) = 0.999999999998\, V_0$$

(2) But if $l = 20$ km:

$$V_{BB'} = 0.91\, V_0$$

$$\phi_0 = \frac{\omega l}{c} = \frac{2\pi f l}{c} = 2\pi \frac{l}{\lambda} \quad \text{radians.} \qquad (2.4)$$

When l/λ is very small, transmission-line effects may be ignored, but when $l/\lambda \gtrsim 0.01$, it may be necessary to account not only for the phase shift due to the time delay, but also for the presence of reflected signals that may have been bounced back by the load toward the generator.

Dispersion

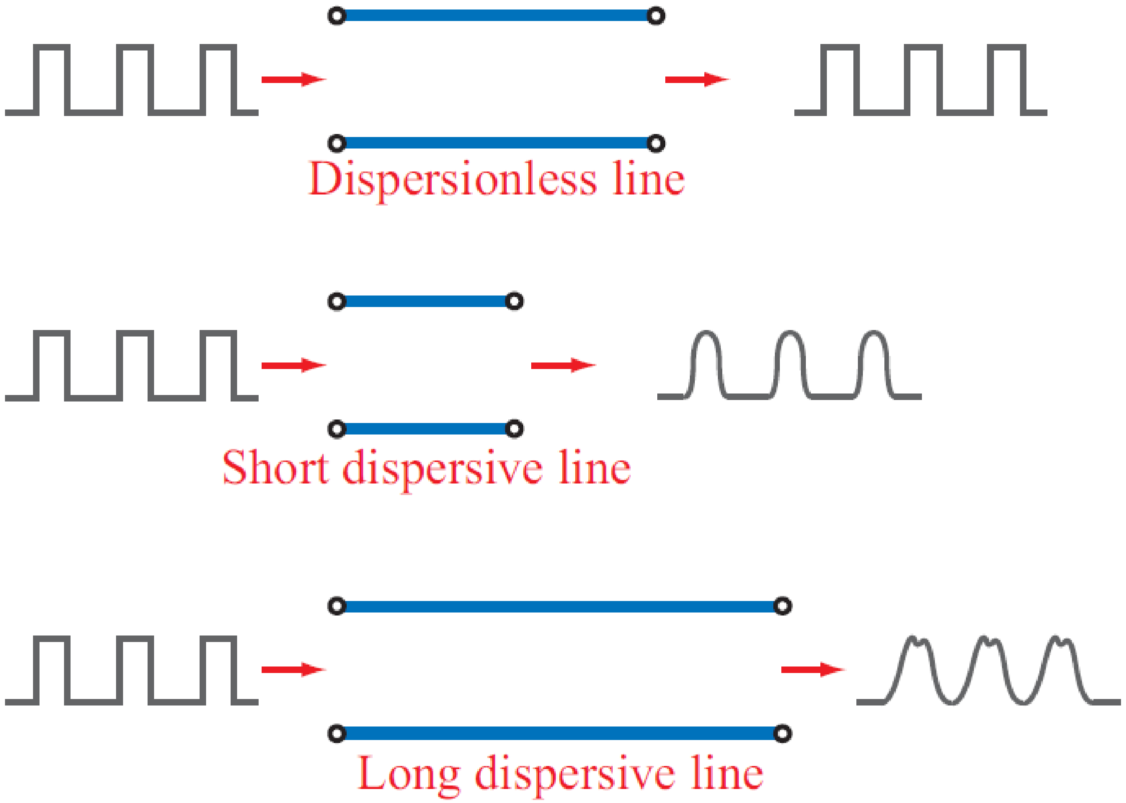
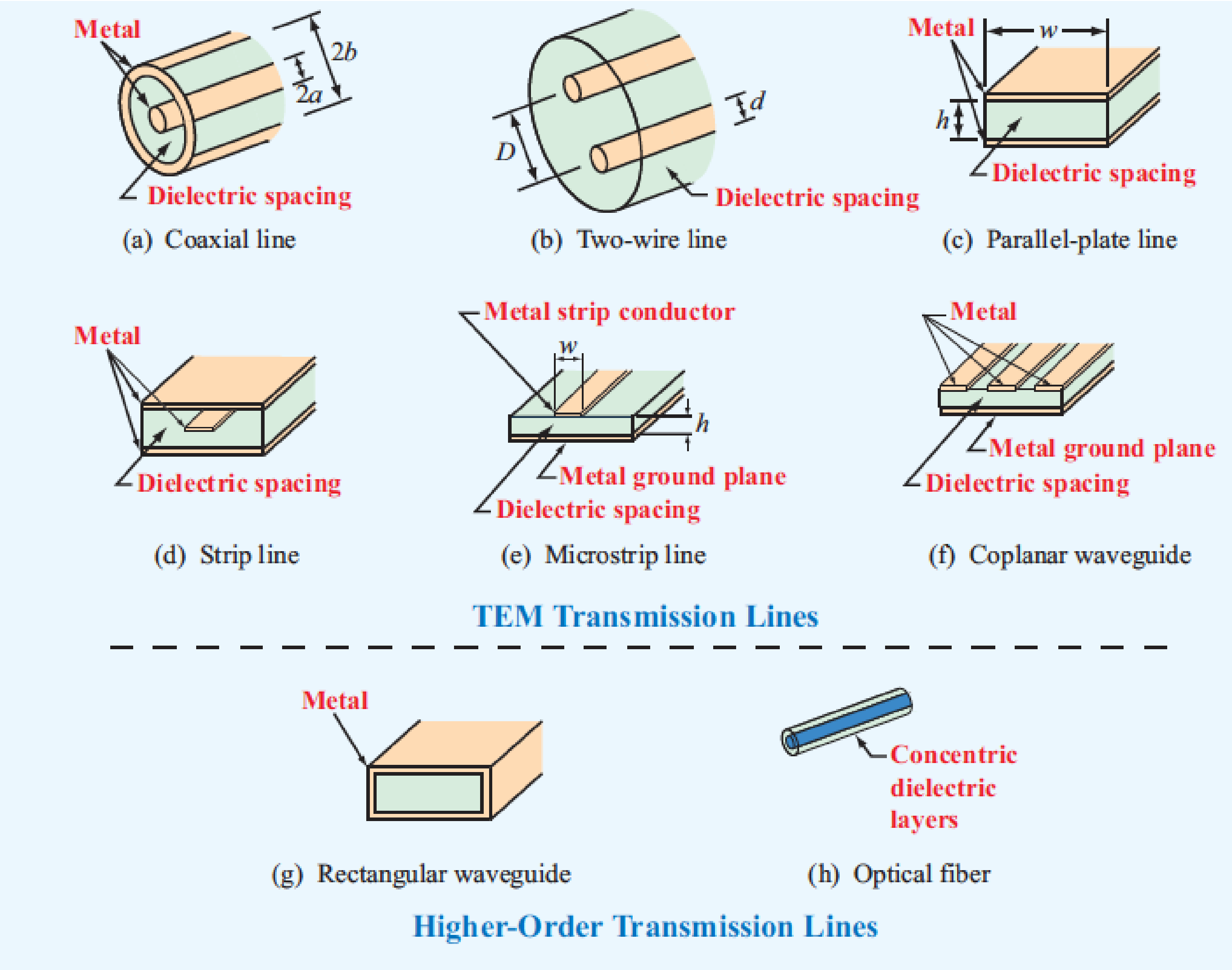


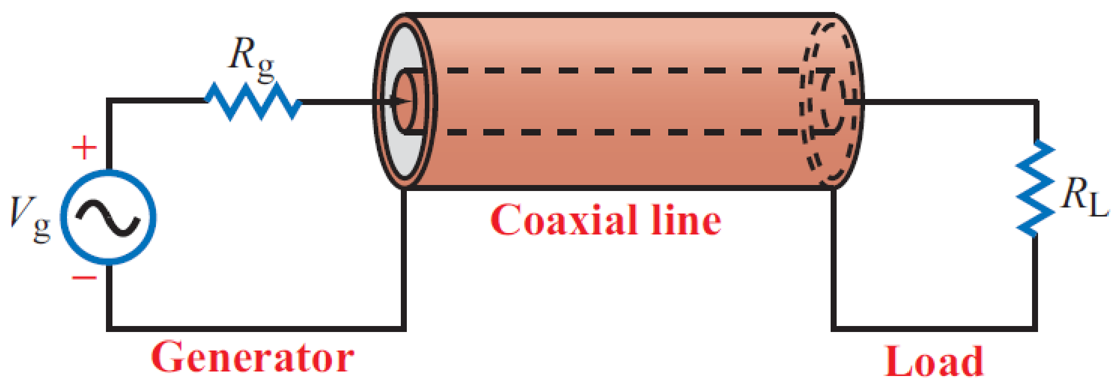
Figure 2-3: A dispersionless line does not distort signals passing through it regardless of its length, whereas a dispersive line distorts the shape of the input pulses because the different frequency components propagate at different velocities. The degree of distortion is proportional to the length of the dispersive line.

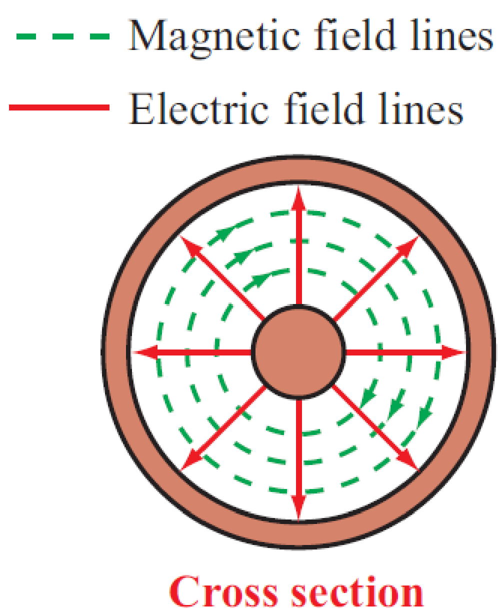
Types of Transmission Modes

TEM (Transverse Electromagnetic): Electric and magnetic fields are orthogonal to one another, and both are orthogonal to direction of propagation



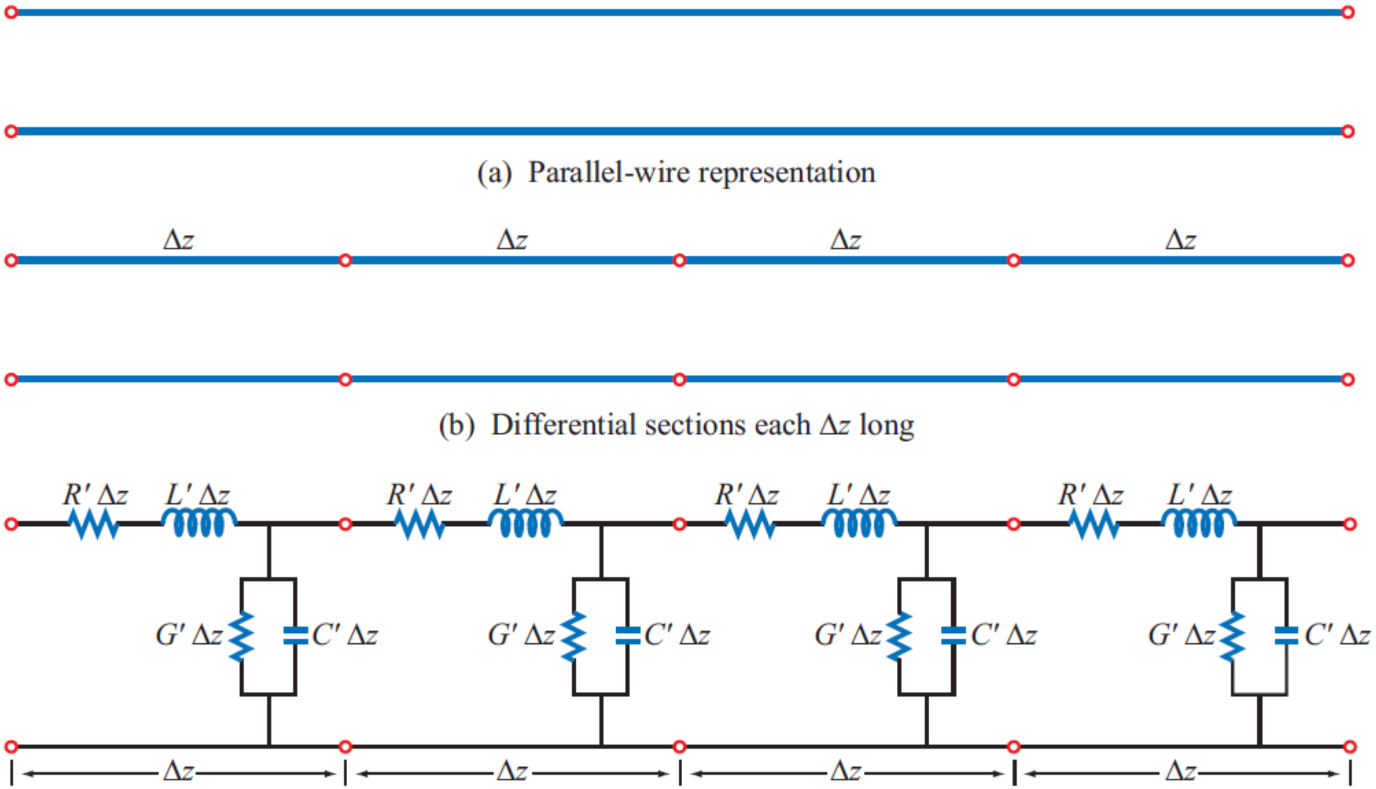
Example of TEM Mode





Electric Field E is radial
Magnetic Field H is azimuthal
Propagation is into the page

Transmission Line Model



(a) Parallel-wire representation

(b) Differential sections each Δz long

(c) Each section is represented by an equivalent circuit

- R': The combined ***resistance*** of both conductors per unit length, in Ω/m,
- L': The combined ***inductance*** of both conductors per unit length, in H/m,
- G': The ***conductance*** of the insulation medium between the two conductors per unit length, in S/m, and
- C': The ***capacitance*** of the two conductors per unit length, in F/m.

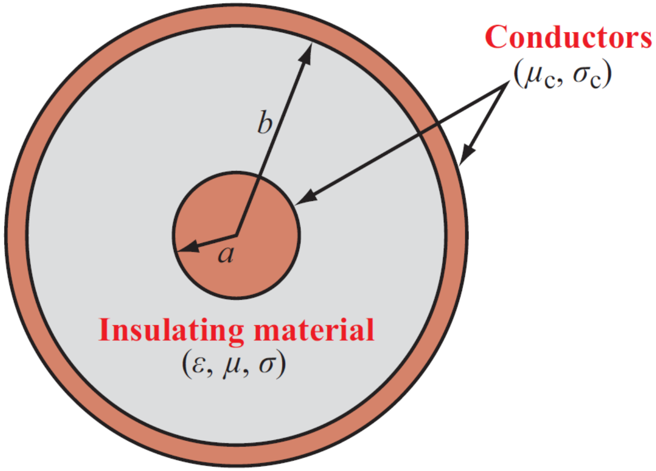
Table 2-1: Transmission-line parameters R', L', G', and C' for three types of lines.

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\varepsilon w}{h}$	F/m

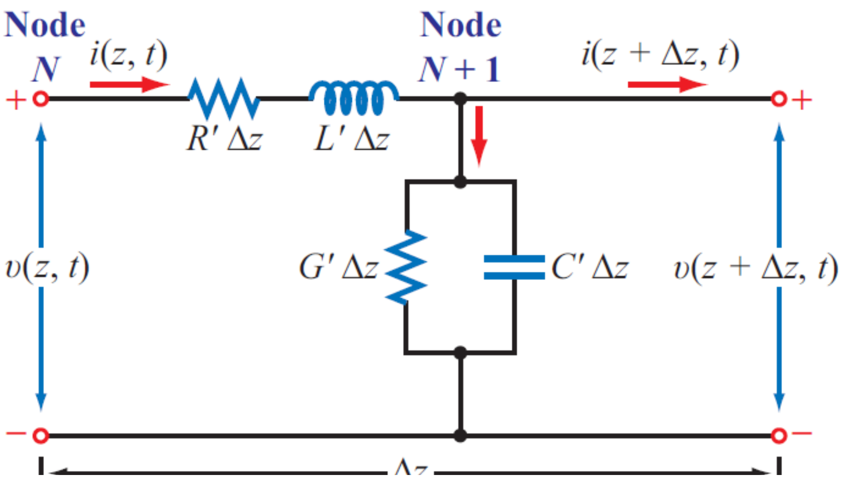
Notes: (1) Refer to Fig. 2-4 for definitions of dimensions. (2) μ, ε, and σ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\pi f \mu_c/\sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(D/d)^2 \gg 1$, then $\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right] \simeq \ln(2D/d)$.

Expressions will be derived in later chapters

*The pertinent **constitutive parameters** apply to all three lines and consist of two groups: (1) μ_c and σ_c are the magnetic permeability and electrical conductivity of the conductors, and (2) ε, μ, and σ are the electrical permittivity, magnetic permeability, and electrical conductivity of the insulation material separating them.*



Transmission-Line Equations



$$\upsilon(z,t) - R'\Delta z\; i(z,t) - L'\Delta z\,\frac{\partial i(z,t)}{\partial t} - \upsilon(z+\Delta z, t) = 0.$$

Upon dividing all terms by Δz and rearranging them, we obtain

$$-\left[\frac{\upsilon(z+\Delta z,t) - \upsilon(z,t)}{\Delta z}\right] = R' i(z,t) + L'\frac{\partial i(z,t)}{\partial t}. \qquad (2.13)$$

In the limit as $\Delta z \to 0$, Eq. (2.13) becomes a differential equation:

$$\boxed{-\frac{\partial \upsilon(z,t)}{\partial z} = R'\; i(z,t) + L'\frac{\partial i(z,t)}{\partial t}. \qquad (2.14)}$$

$$i(z,t) - G'\Delta z\;\upsilon(z+\Delta z, t) - C'\Delta z\,\frac{\partial \upsilon(z+\Delta z,t)}{\partial t} - i(z+\Delta z, t) = 0. \qquad (2.15)$$

Upon dividing all terms by Δz and taking the limit $\Delta z \to 0$, Eq. (2.15) becomes a second-order differential equation:

$$\boxed{-\frac{\partial i(z,t)}{\partial z} = G'\;\upsilon(z,t) + C'\frac{\partial \upsilon(z,t)}{\partial t}. \qquad (2.16)}$$

ac signals: use phasors

$$\upsilon(z,t) = \mathfrak{Re}[\widetilde{V}(z)\, e^{j\omega t}],$$
$$i(z,t) = \mathfrak{Re}[\widetilde{I}(z)\, e^{j\omega t}],$$

$$\boxed{\begin{aligned} -\frac{d\widetilde{V}(z)}{dz} &= (R' + j\omega L')\,\widetilde{I}(z), \\ -\frac{d\widetilde{I}(z)}{dz} &= (G' + j\omega C')\,\widetilde{V}(z). \end{aligned}}$$

Telegrapher's equations

Derivation of Wave Equations

$$-\frac{d\widetilde{V}(z)}{dz} = (R' + j\omega L')\,\widetilde{I}(z),$$
$$-\frac{d\widetilde{I}(z)}{dz} = (G' + j\omega C')\,\widetilde{V}(z).$$

Combining the two equations leads to:

$$\frac{d^2\widetilde{V}(z)}{dz^2} - (R' + j\omega L')(G' + j\omega C')\,\widetilde{V}(z) = 0,$$

$$\frac{d^2\widetilde{V}(z)}{dz^2} - \gamma^2\,\widetilde{V}(z) = 0, \qquad (2.21)$$

Second-order differential equation

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}\,. \qquad (2.22)$$

complex propagation constant

attenuation constant

Phase constant

$$\gamma = \alpha + j\beta$$

$$\alpha = \Re\mathfrak{e}(\gamma) = \Re\mathfrak{e}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad \text{(Np/m)}, \qquad (2.25a)$$

$$\beta = \Im\mathfrak{m}(\gamma) = \Im\mathfrak{m}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad \text{(rad/m)}. \qquad (2.25b)$$

Solution of Wave Equations (cont.)

$$\frac{d^2\widetilde{V}(z)}{dz^2} - \gamma^2\,\widetilde{V}(z) = 0, \qquad (2.21)$$

$$\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2\,\tilde{I}(z) = 0. \qquad (2.23)$$

Proposed form of solution:

$$\widetilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)},$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad \text{(A)}.$$

Using:

$$-\frac{d\widetilde{V}(z)}{dz} = (R' + j\omega L')\,\tilde{I}(z),$$

It follows that:

$$\tilde{I}(z) = \frac{\gamma}{R' + j\omega L'}\left[V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right]$$

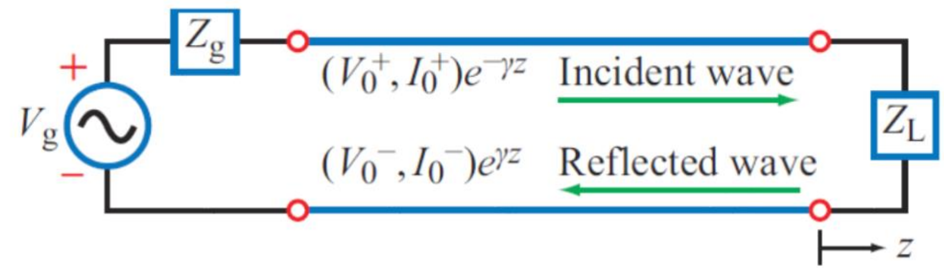


Figure 2-9: In general, a transmission line can support two traveling waves, an incident wave [with voltage and current amplitudes (V_0^+, I_0^+)] traveling along the $+z$-direction (towards the load) and a reflected wave [with (V_0^-, I_0^-)] traveling along the $-z$-direction (towards the source).

Comparison of each term with the corresponding term in Eq. (2.26b) leads us to conclude that

$$\frac{V_0^+}{I_0^+} = Z_0 = \frac{-V_0^-}{I_0^-}, \qquad (2.28)$$

where

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \quad (\Omega), \qquad (2.29)$$

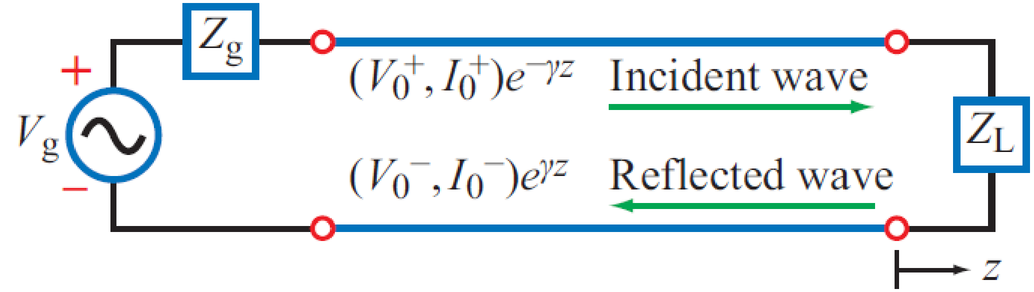
is called the ***characteristic impedance*** of the line.

Solution of Wave Equations (cont.)

In general:

$$V_0^+ = |V_0^+|e^{j\phi^+},$$

$$V_0^- = |V_0^-|e^{j\phi^-}.$$



The presence of two waves on the line propagating in opposite directions produces a ***standing wave****.*

$$\begin{aligned}
\upsilon(z,t) &= \mathfrak{Re}(\widetilde{V}(z)e^{j\omega t}) \\
&= \mathfrak{Re}\left[\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right) e^{j\omega t}\right] \\
&= \mathfrak{Re}[|V_0^+|e^{j\phi^+} e^{j\omega t} e^{-(\alpha+j\beta)z} \\
&\qquad + |V_0^-|e^{j\phi^-} e^{j\omega t} e^{(\alpha+j\beta)z}] \\
&= |V_0^+|e^{-\alpha z} \cos(\omega t - \beta z + \phi^+) \\
&\qquad + |V_0^-|e^{\alpha z} \cos(\omega t + \beta z + \phi^-)
\end{aligned}$$

wave along $+z$ because coefficients of t and z have opposite signs

wave along $-z$ because coefficients of t and z have the same sign

Example 2-1: Air Line

An *air line* is a transmission line in which air separates the two conductors, which renders $G' = 0$ because $\sigma = 0$. In addition, assume that the conductors are made of a material with high conductivity so that $R' \simeq 0$. For an air line with a characteristic impedance of 50 Ω and a phase constant of 20 rad/m at 700 MHz, find the line inductance L' and the line capacitance C'.

Solution: The following quantities are given:

$$Z_0 = 50\ \Omega, \qquad \beta = 20\ \text{rad/m},$$
$$f = 700\ \text{MHz} = 7 \times 10^8\ \text{Hz}.$$

With $R' = G' = 0$, Eqs. (2.25b) and (2.29) reduce to

$$\beta = \mathfrak{Im}\left[\sqrt{(j\omega L')(j\omega C')}\right] = \mathfrak{Im}\left(j\omega\sqrt{L'C'}\right) = \omega\sqrt{L'C'},$$

$$Z_0 = \sqrt{\frac{j\omega L'}{j\omega C'}} = \sqrt{\frac{L'}{C'}}.$$

The ratio of β to Z_0 is

$$\frac{\beta}{Z_0} = \omega C',$$

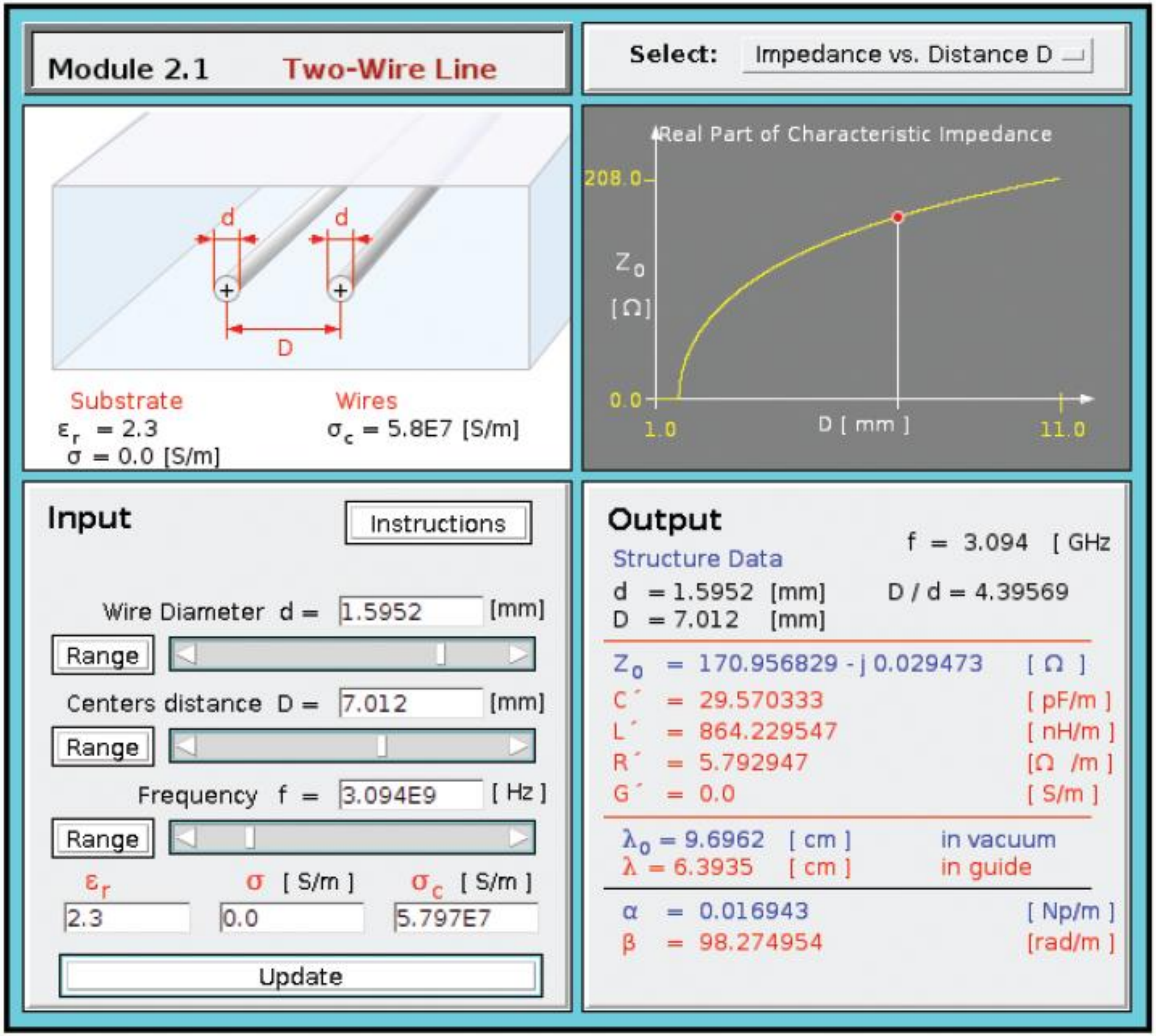
or

$$C' = \frac{\beta}{\omega Z_0} = \frac{20}{2\pi \times 7 \times 10^8 \times 50} = 9.09 \times 10^{-11}\ \text{(F/m)} = \boxed{90.9\ \text{(pF/m)}.}$$

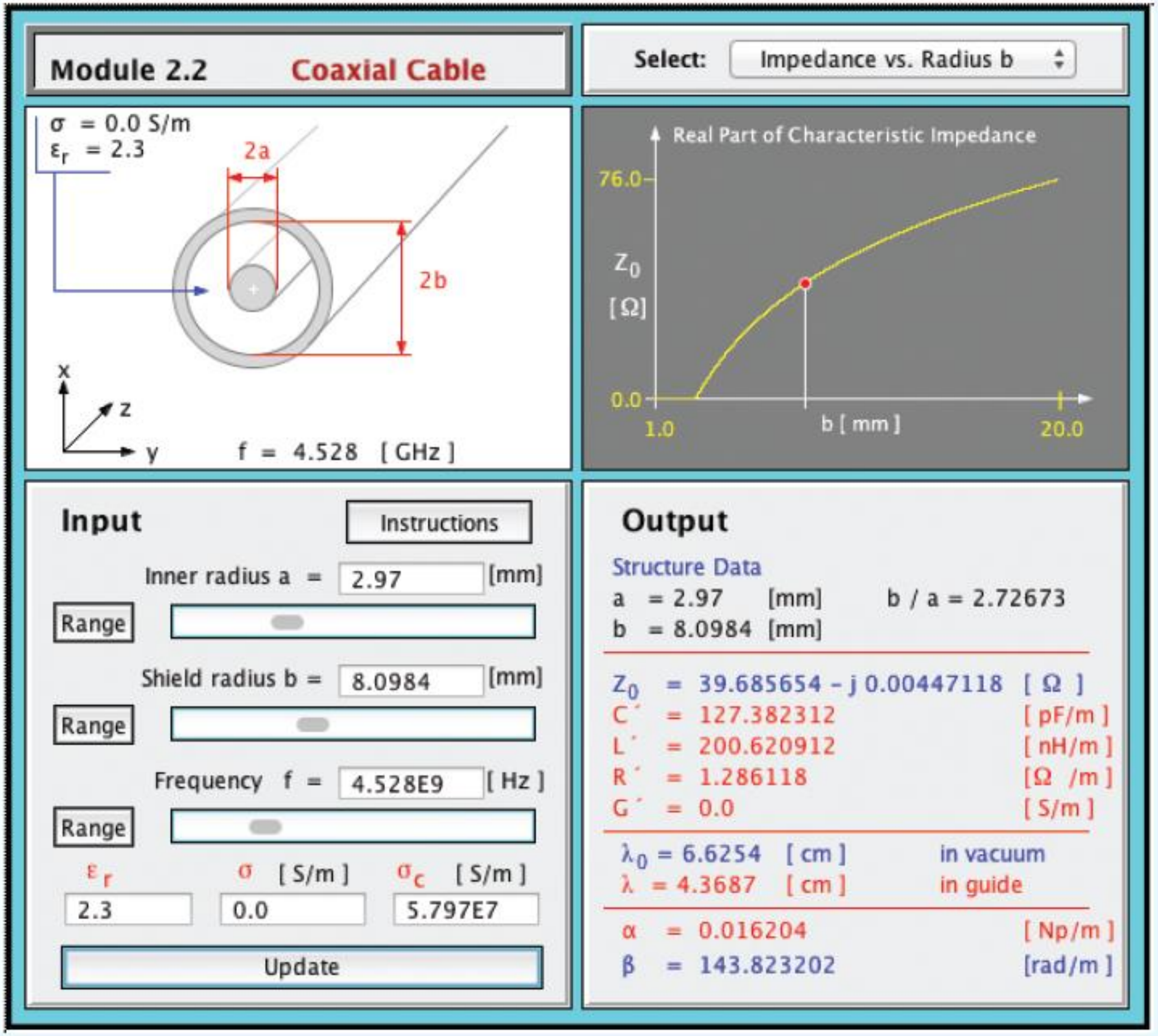
From $Z_0 = \sqrt{L'/C'}$, it follows that

$$L' = Z_0^2 C' = (50)^2 \times 90.9 \times 10^{-12} = 2.27 \times 10^{-7}\ \text{(H/m)} = \boxed{227\ \text{(nH/m)}.}$$

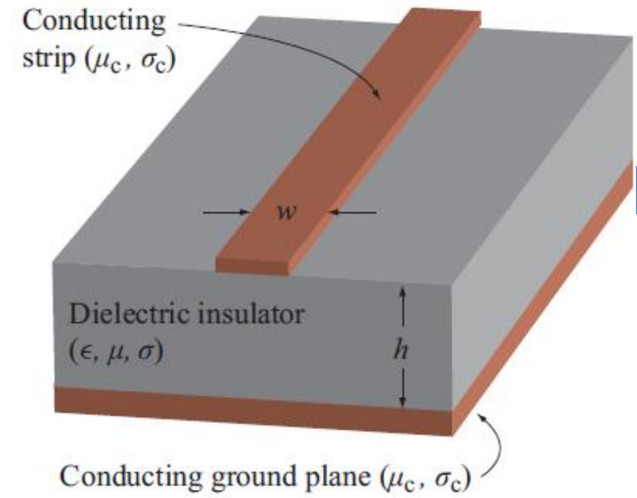
Module 2.1 Two-Wire Line The input data specifies the geometric and electric parameters of a two-wire transmission line. The output includes the calculated values for the line parameters, characteristic impedance Z_0, and attenuation and phase constants, as well as plots of Z_0 as a function of d and D.



Module 2.2 Coaxial Cable Except for changing the geometric parameters to those of a coaxial transmission line, this module offers the same output information as Module 2.1.



Lossless Microstrip Line

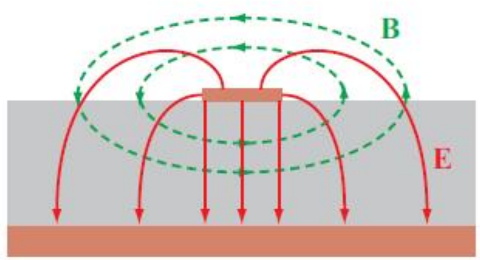


(a) Longitudinal view

Phase velocity in dielectric: $u_p = \dfrac{c}{\sqrt{\varepsilon_r}}$

Phase velocity for microstrip: $u_p = \dfrac{c}{\sqrt{\varepsilon_{eff}}}$

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \left(\frac{\varepsilon_r - 1}{2}\right)\left(1 + \frac{10}{s}\right)^{-xy}, \qquad (2.36)$$



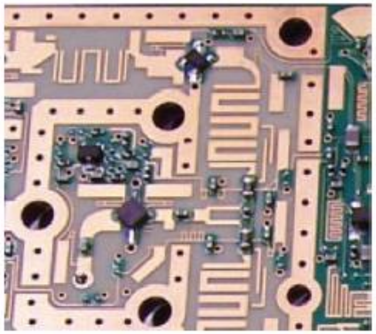
(b) Cross-sectional view with **E** and **B** field lines

where s is the width-to-thickness ratio,

Quasi-TEM

$$s = \frac{w}{h}, \qquad (2.37)$$

and x and y are intermediate variables given by


(c) Microwave circuit

$$x = 0.56\left[\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3}\right]^{0.05}, \qquad (2.38a)$$

$$y = 1 + 0.02\ln\left(\frac{s^4 + 3.7 \times 10^{-4}s^2}{s^4 + 0.43}\right) + 0.05\ln(1 + 1.7 \times 10^{-4}s^3). \qquad (2.38b)$$

Microstrip (cont.)

The characteristic impedance of the microstrip line is given by

$$Z_0 = \frac{60}{\sqrt{\varepsilon_{\text{eff}}}} \ln\left\{\frac{6+(2\pi-6)e^{-t}}{s} + \sqrt{1+\frac{4}{s^2}}\right\}, \qquad (2.39)$$

$$t = \left(\frac{30.67}{s}\right)^{0.75}$$

$$R' = 0 \qquad (\text{because } \sigma_c = \infty),$$

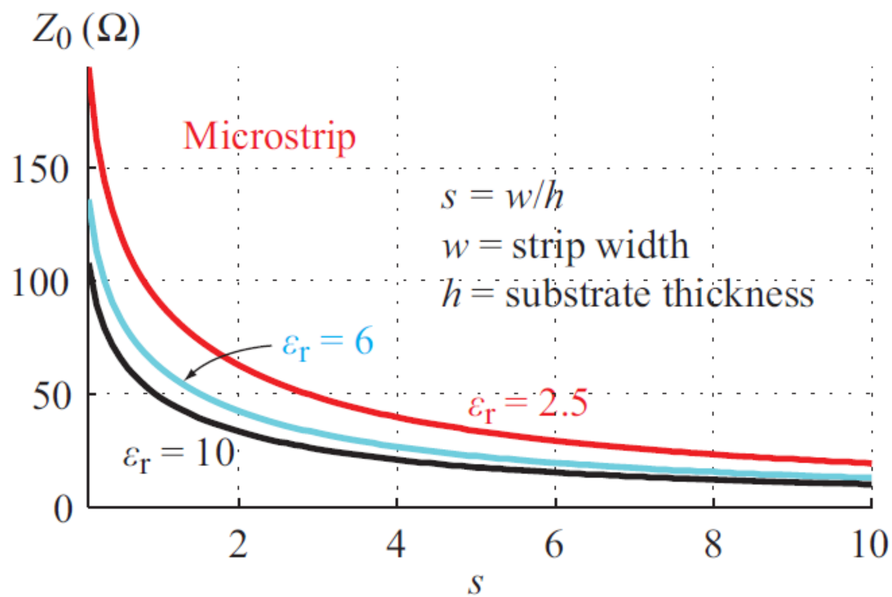
$$G' = 0 \qquad (\text{because } \sigma = 0),$$

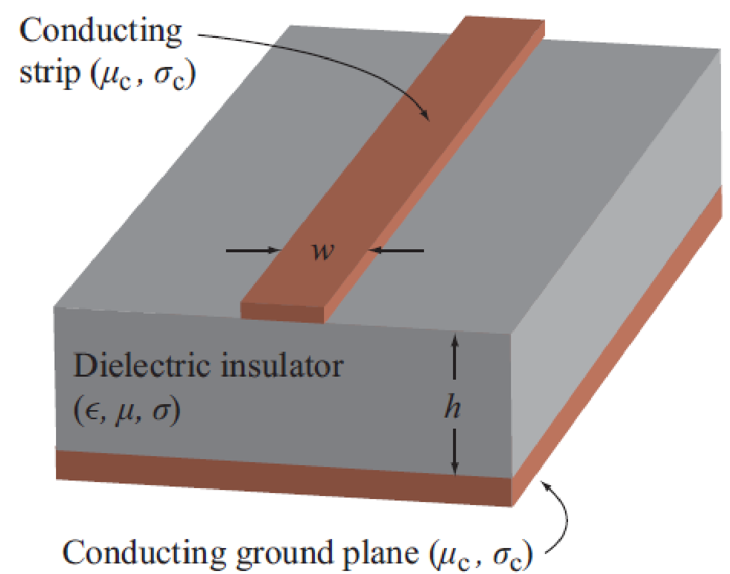
$$C' = \frac{\sqrt{\varepsilon_{\text{eff}}}}{Z_0 c},$$

$$L' = Z_0^2 C',$$

$$\alpha = 0 \qquad (\text{because } R' = G' = 0),$$

$$\beta = \frac{\omega}{c}\sqrt{\varepsilon_{\text{eff}}}.$$





(a) Longitudinal view

Microstrip (cont.)

Inverse process:

Given Z_0, find s

The solution formulas are based on two numerical fits, defined in terms of the value of Z_0 relative to that of the effective permittivity.

(a) For $Z_0 \leq (44 - 2\varepsilon_r)\ \Omega$,

$$s = \frac{w}{h} = \frac{2}{\pi}\left\{(q-1) - \ln(2q-1) + \frac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(q-1) + 0.29 - \frac{0.52}{\varepsilon_r}\right]\right\} \tag{2.42a}$$

with

$$q = \frac{60\pi^2}{Z_0\sqrt{\varepsilon_r}}, \tag{2.42b}$$

and

(b) for $Z_0 \geq (44 - 2\varepsilon_r)\ \Omega$,

$$s = \frac{w}{h} = \frac{8e^p}{e^{2p} - 2}, \tag{2.43a}$$

with

$$p = \sqrt{\frac{\varepsilon_r + 1}{2}}\,\frac{Z_0}{60} + \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left(0.23 + \frac{0.12}{\varepsilon_r}\right). \tag{2.43b}$$

Example 2-2: Microstrip Line

A 50-Ω microstrip line uses a 0.5-mm–thick sapphire substrate with $\varepsilon_r = 9$. What is the width of its copper strip?

Solution: Since $Z_0 = 50 > 44 - 18 = 32$, we should use Eq. (2.43):

$$p = \sqrt{\frac{\varepsilon_r + 1}{2}} \times \frac{Z_0}{60} + \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left(0.23 + \frac{0.12}{\varepsilon_r}\right)$$

$$= \sqrt{\frac{9+1}{2}} \times \frac{50}{60} + \left(\frac{9-1}{9+1}\right)\left(0.23 + \frac{0.12}{9}\right)$$

$$= 2.06,$$

$$s = \frac{w}{h} = \frac{8e^p}{e^{2p} - 2} = \frac{8e^{2.06}}{e^{4.12} - 2} = 1.056.$$

Hence,

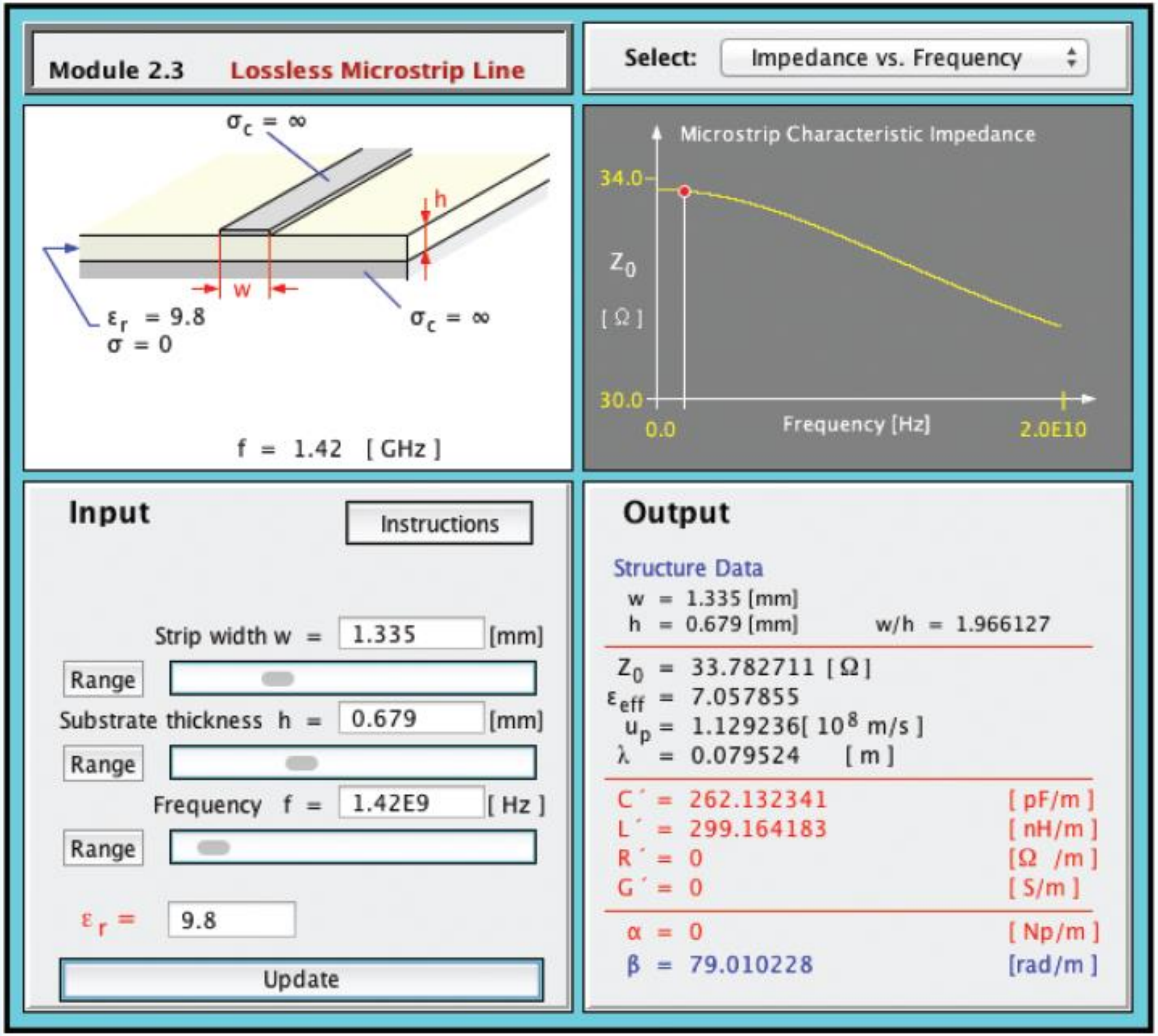
$$w = sh = 1.056 \times 0.5 \text{ mm} = \boxed{0.53 \text{ mm.}}$$

To check our calculations, we will use $s = 1.056$ to calculate Z_0 to verify that the value we obtained is indeed equal or close to 50 Ω. With $\varepsilon_r = 9$, Eqs. (2.36) to (2.40) yield

$$x = 0.55,$$
$$y = 0.99,$$
$$t = 12.51,$$
$$\varepsilon_{\text{eff}} = 6.11,$$
$$Z_0 = 49.93\ \Omega.$$

The calculated value of Z_0 is, for all practical purposes, equal to the value specified in the problem statement.

Module 2.3 Lossless Microstrip Line The output panel lists the values of the transmission-line parameters and displays the variation of Z_0 and ϵ_{eff} with h and w.



Lossless Transmission Line

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}\,.$$

If $\quad R' \ll \omega L'$ and $G' \ll \omega C'$

Then:

$$\gamma = \alpha + j\beta = j\omega\sqrt{L'C'}\,, \qquad (2.44)$$

which in turn implies that

$$\alpha = 0 \qquad \text{(lossless line)},$$
$$\beta = \omega\sqrt{L'C'} \qquad \text{(lossless line)}. \qquad (2.45)$$

For the characteristic impedance, application of the lossless line conditions to Eq. (2.29) leads to

$$Z_0 = \sqrt{\frac{L'}{C'}} \qquad \text{(lossless line)}, \qquad (2.46)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{L'C'}}\,,$$

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}\,.$$

$$\beta = \omega\sqrt{\mu\varepsilon} \qquad \text{(rad/m)}, \qquad (2.49)$$
$$u_p = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \text{(m/s)}, \qquad (2.50)$$

If sinusoidal waves of different frequencies travel on a transmission line with the same phase velocity, the line is called ***nondispersive***.

$$\lambda = \frac{u_p}{f} = \frac{c}{f}\,\frac{1}{\sqrt{\varepsilon_r}} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

Table 2-2: Characteristic parameters of transmission lines.

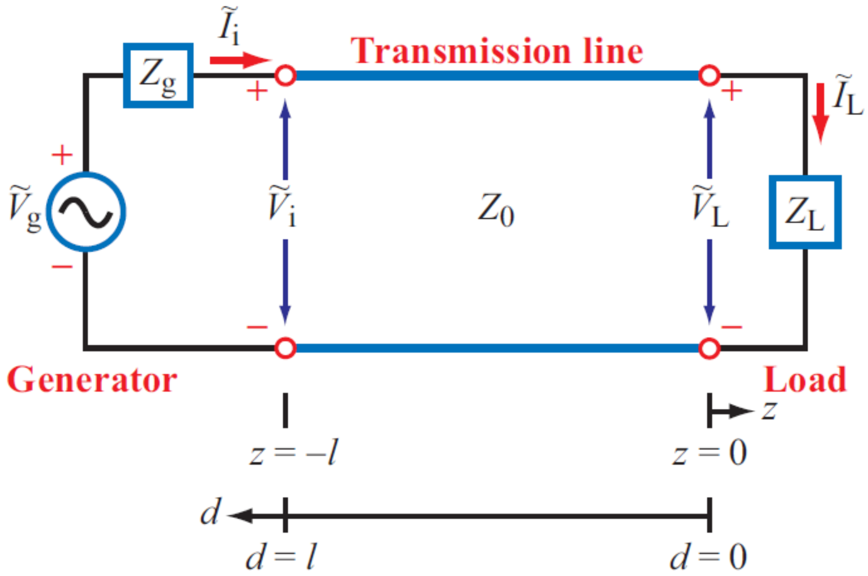
	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\dfrac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless $(R' = G' = 0)$	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \left(60/\sqrt{\varepsilon_r}\right)\ln(b/a)$
Lossless two-wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \left(120/\sqrt{\varepsilon_r}\right) \cdot \ln[(D/d) + \sqrt{(D/d)^2 - 1}]$ $Z_0 \simeq \left(120/\sqrt{\varepsilon_r}\right)\ln(2D/d)$, if $D \gg d$
Lossless parallel-plate	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \left(120\pi/\sqrt{\varepsilon_r}\right)(h/w)$

Notes: (1) $\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $c = 1/\sqrt{\mu_0\varepsilon_0}$, and $\sqrt{\mu_0/\varepsilon_0} \simeq (120\pi)\ \Omega$, where ε_r is the relative permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer conductors. (3) For two-wire line, d = wire diameter and D = separation between wire centers. (4) For parallel-plate line, w = width of plate and h = separation between the plates.

Voltage Reflection Coefficient

$$\tilde{V}_L = \tilde{V}(z=0) = V_0^+ + V_0^-,$$

$$\tilde{I}_L = \tilde{I}(z=0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}.$$



At the load ($z = 0$):

$$Z_L = \frac{\tilde{V}_L}{\tilde{I}_L}$$

$$\tilde{V}_L = \tilde{V}(z=0) = V_0^+ + V_0^-,$$

$$\tilde{I}_L = \tilde{I}(z=0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}.$$

Using these expressions in Eq. (2.55), we obtain

$$Z_L = \left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right) Z_0.$$

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z_L - 1}{z_L + 1} \quad \text{(dimensionless)}, \qquad (2.59)$$

Reflection coefficient

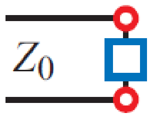
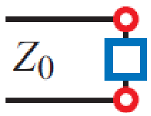
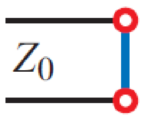
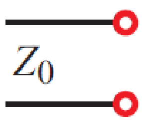
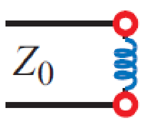
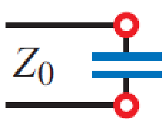
$$z_L = \frac{Z_L}{Z_0}$$

Normalized load impedance

Voltage Reflection Coefficient

$$\Gamma = |\Gamma| e^{j\theta_r}$$

Reflection Coefficient $\Gamma = |\Gamma| e^{j\theta_r}$

Load	$\lvert\Gamma\rvert$	θ_r
 $Z_L = (r + jx)Z_0$	$\left[\dfrac{(r-1)^2 + x^2}{(r+1)^2 + x^2}\right]^{1/2}$	$\tan^{-1}\left(\dfrac{x}{r-1}\right) - \tan^{-1}\left(\dfrac{x}{r+1}\right)$
 Z_0	0 (no reflection)	irrelevant
 (short)	1	$\pm 180°$ (phase opposition)
 (open)	1	0 (in-phase)
 $jX = j\omega L$	1	$\pm 180° - 2\tan^{-1} x$
 $jX = \dfrac{-j}{\omega C}$	1	$\pm 180° + 2\tan^{-1} x$

$$z_L = Z_L / Z_0 = (R + jX)/Z_0 = r + jx$$

Current Reflection Coefficient

$$\frac{I_0^-}{I_0^+} = -\frac{V_0^-}{V_0^+} = -\Gamma. \qquad (2.61)$$

We note that whereas the ratio of the voltage amplitudes is equal to Γ, the ratio of the current amplitudes is equal to $-\Gamma$.

Example 2-3: Reflection Coefficient of a Series *RC* Load

A 100-Ω transmission line is connected to a load consisting of a 50-Ω resistor in series with a 10-pF capacitor. Find the reflection coefficient at the load for a 100-MHz signal.

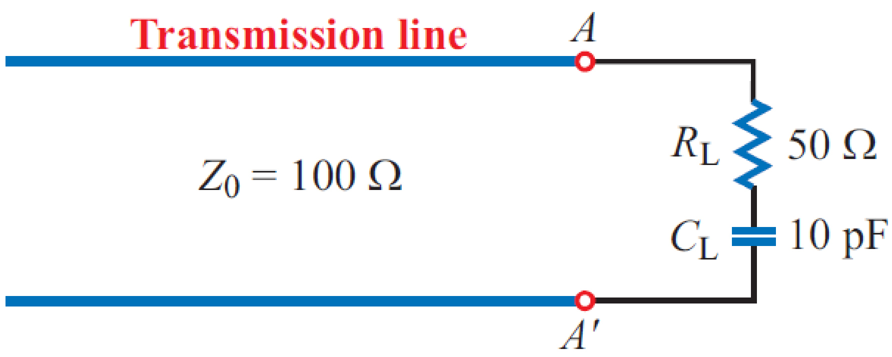
Solution: The following quantities are given (Fig. 2-13):

$$R_L = 50\ \Omega, \qquad C_L = 10\text{ pF} = 10^{-11}\text{ F},$$

$$Z_0 = 100\ \Omega, \qquad f = 100\text{ MHz} = 10^8\text{ Hz}.$$

The normalized load impedance is

$$\begin{aligned} z_L = \frac{Z_L}{Z_0} &= \frac{R_L - j/(\omega C_L)}{Z_0} \\ &= \frac{1}{100}\left(50 - j\frac{1}{2\pi \times 10^8 \times 10^{-11}}\right) \\ &= (0.5 - j1.59)\ \Omega. \end{aligned}$$



$$\begin{aligned} \Gamma &= \frac{z_L - 1}{z_L + 1} \\ &= \frac{0.5 - j1.59 - 1}{0.5 - j1.59 + 1} \\ &= \frac{-0.5 - j1.59}{1.5 - j1.59} = \frac{-1.67e^{j72.6^\circ}}{2.19e^{-j46.7^\circ}} = -0.76e^{j119.3^\circ}. \end{aligned}$$

This result may be converted into the form of Eq. (2.62) by replacing the minus sign with e^{-j180°. Thus,

$$\Gamma = 0.76e^{j119.3^\circ}e^{-j180^\circ} = 0.76e^{-j60.7^\circ} = 0.76\angle{-60.7^\circ},$$

or

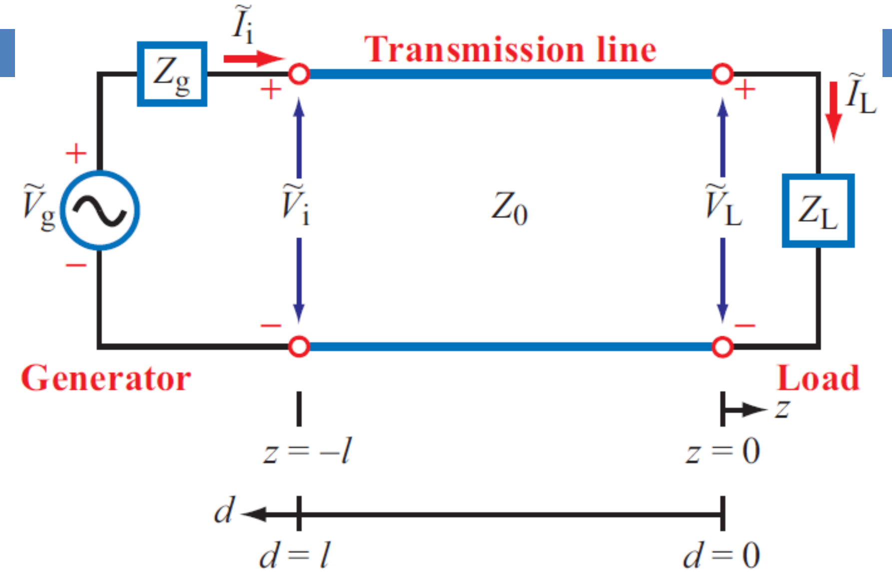
$$\boxed{|\Gamma| = 0.76, \qquad \theta_r = -60.7^\circ.}$$

Standing Waves

Using the relation $V_0^- = \Gamma V_0^+$ yields

$$\tilde{V}(z) = V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z}),$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma e^{j\beta z}).$$



$$|\tilde{V}(z)| = \left\{\left[V_0^+(e^{-j\beta z} + |\Gamma|e^{j\theta_r}e^{j\beta z})\right] \cdot \left[(V_0^+)^*(e^{j\beta z} + |\Gamma|e^{-j\theta_r}e^{-j\beta z})\right]\right\}^{1/2}$$

$$= |V_0^+|\left[1 + |\Gamma|^2 + |\Gamma|(e^{j(2\beta z+\theta_r)} + e^{-j(2\beta z+\theta_r)})\right]^{1/2}$$

$$= |V_0^+|\left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta z + \theta_r)\right]^{1/2}, \qquad (2.64)$$

To express the magnitude of $\tilde{V}$ as a function of d instead of z, we replace z with $-d$ on the right-hand side of Eq. (2.64):

$$|\tilde{V}(d)| = |V_0^+|\left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_r)\right]^{1/2}. \qquad (2.66)$$

voltage magnitude

By applying the same steps to Eq. (2.63b), a similar expression can be derived for $|\tilde{I}(d)|$, the magnitude of the current $\tilde{I}(d)$:

$$|\tilde{I}(d)| = \frac{|V_0^+|}{Z_0}\left[1 + |\Gamma|^2 - 2|\Gamma|\cos(2\beta d - \theta_r)\right]^{1/2}. \qquad (2.67)$$

current magnitude

Standing-Wave Pattern

Whereas the repetition period is λ for the incident and reflected waves considered individually, the repetition period of the standing-wave pattern is λ/2.

$$|\tilde{V}(d)| = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta d - \theta_r)\right]^{1/2}. \quad (2.66)$$

$$|\tilde{I}(d)| = \frac{|V_0^+|}{Z_0} \left[1 + |\Gamma|^2 - 2|\Gamma| \cos(2\beta d - \theta_r)\right]^{1/2}. \quad (2.67)$$

Voltage magnitude is maximum when $(2\beta d_{min} - \theta_r) = (2n+1)\pi$

When voltage is a maximum, current is a minimum, and vice versa

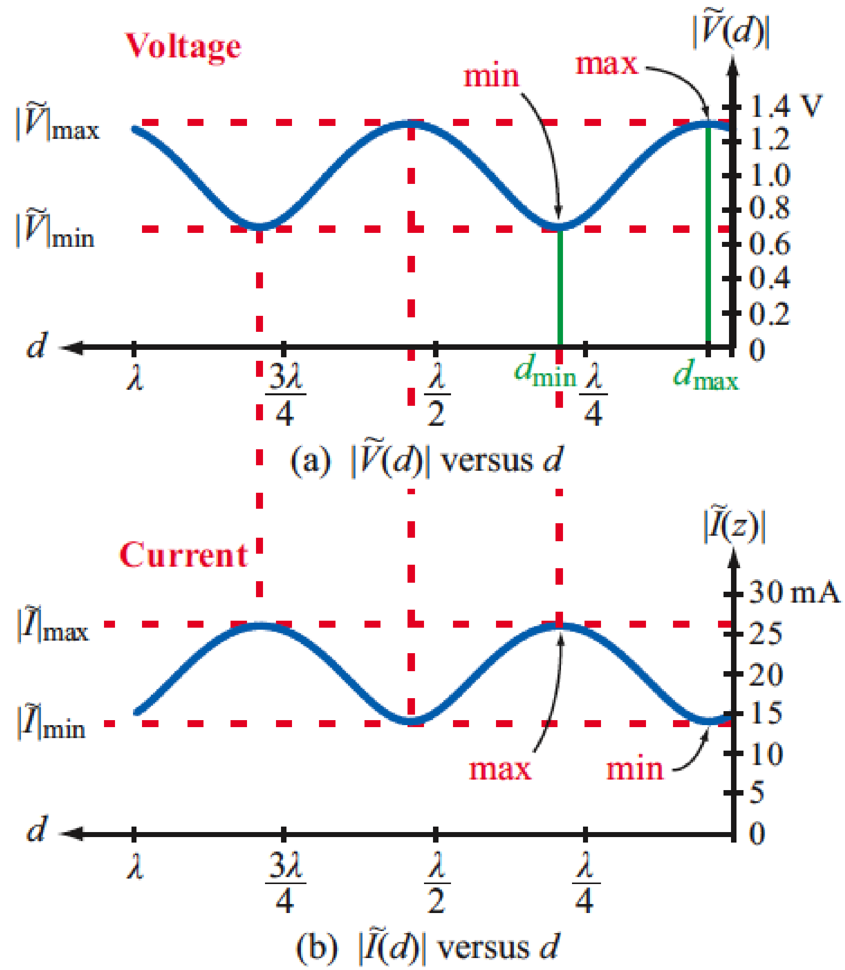
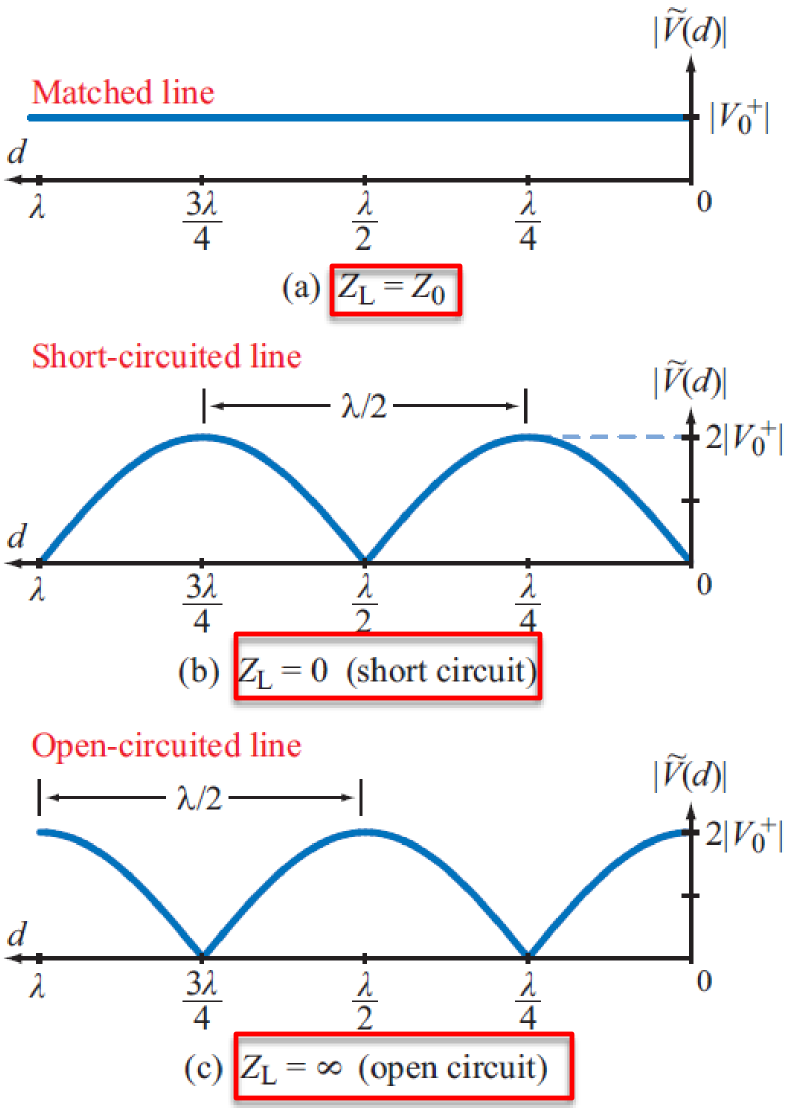


Figure 2-14: Standing-wave pattern for (a) $|\tilde{V}(d)|$ and (b) $|\tilde{I}(d)|$ for a lossless transmission line of characteristic impedance $Z_0 = 50\ \Omega$, terminated in a load with a reflection coefficient $\Gamma = 0.3e^{j30°}$. The magnitude of the incident wave $|V_0^+| = 1$ V. The standing-wave ratio is $S = |\tilde{V}|_{max}/|\tilde{V}|_{min} = 1.3/0.7 = 1.86$.

Standing Wave Patterns for 3 Types of Loads



With no reflected wave present, there will be no interference and no standing waves.

Example 2-4: $|\Gamma|$ for Purely Reactive Load

Show that $|\Gamma| = 1$ for a lossless line connected to a purely reactive load.

Solution: The load impedance of a purely reactive load is

$$Z_L = jX_L.$$

From Eq. (2.59), the reflection coefficient is

$$\begin{aligned}\Gamma &= \frac{Z_L - Z_0}{Z_L + Z_0} \\ &= \frac{jX_L - Z_0}{jX_L + Z_0} \\ &= \frac{-(Z_0 - jX_L)}{(Z_0 + jX_L)} = \frac{-\sqrt{Z_0^2 + X_L^2}\, e^{-j\theta}}{\sqrt{Z_0^2 + X_L^2}\, e^{j\theta}} = -e^{-j2\theta},\end{aligned}$$

where $\theta = \tan^{-1} X_L/Z_0$. Hence

$$|\Gamma| = |-e^{-j2\theta}| = [(e^{-j2\theta})(e^{-j2\theta})^*]^{1/2} = 1.$$

Maxima & Minima

Standing-Wave Pattern

Let us denote d_{max} as the distance from the load at which $|\widetilde{V}(d)|$ is a maximum. It then follows that

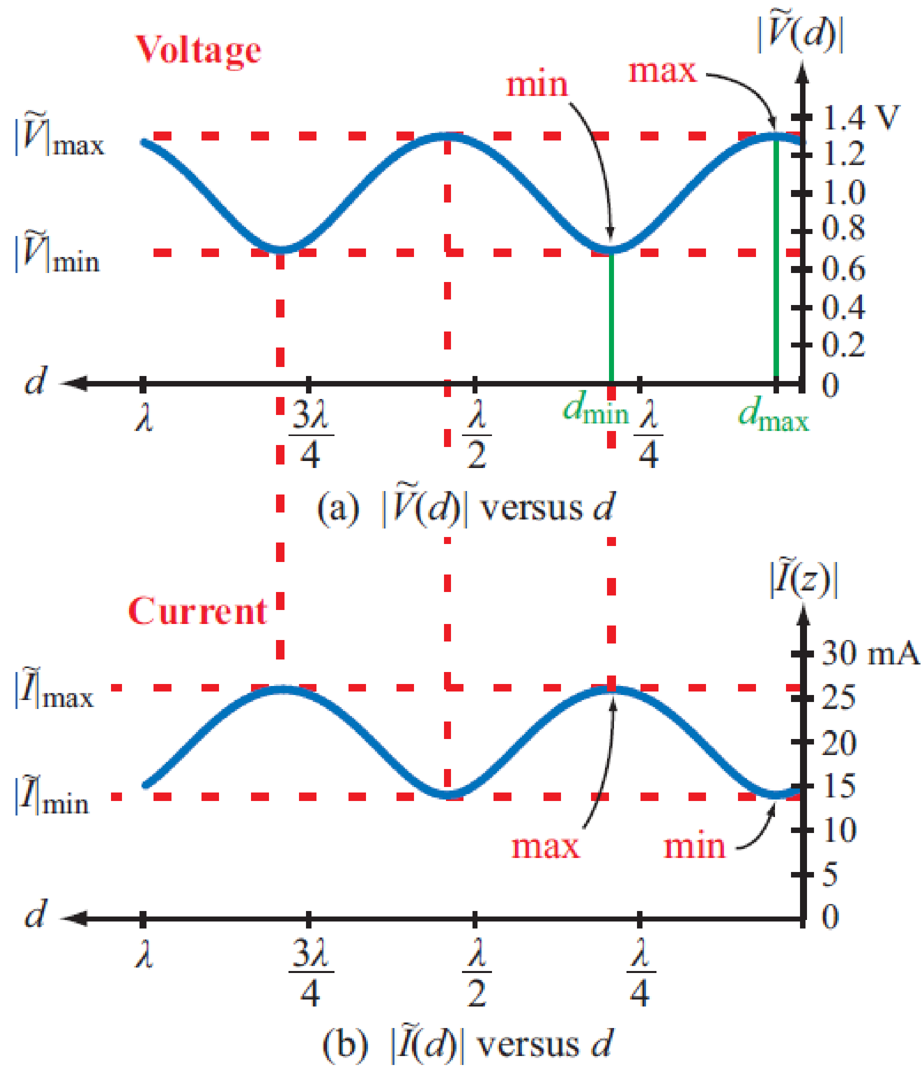
$$|\widetilde{V}(d)| = |\widetilde{V}|_{max} = |V_0^+|[1 + |\Gamma|], \qquad (2.68)$$

when

$$2\beta d_{max} - \theta_r = 2n\pi, \qquad (2.69)$$

with $n = 0$ or a positive integer. Solving Eq. (2.69) for d_{max}, we have

$$d_{max} = \frac{\theta_r + 2n\pi}{2\beta} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2}, \quad \begin{cases} n = 1, 2, \ldots & \text{if } \theta_r < 0, \\ n = 0, 1, 2, \ldots & \text{if } \theta_r \geq 0, \end{cases} \qquad (2.70)$$



(a) $|\widetilde{V}(d)|$ versus d

(b) $|\widetilde{I}(d)|$ versus d

Maxima & Minima (cont.)

$$|\widetilde{V}|_{\min} = |V_0^+|[1 - |\Gamma|],$$

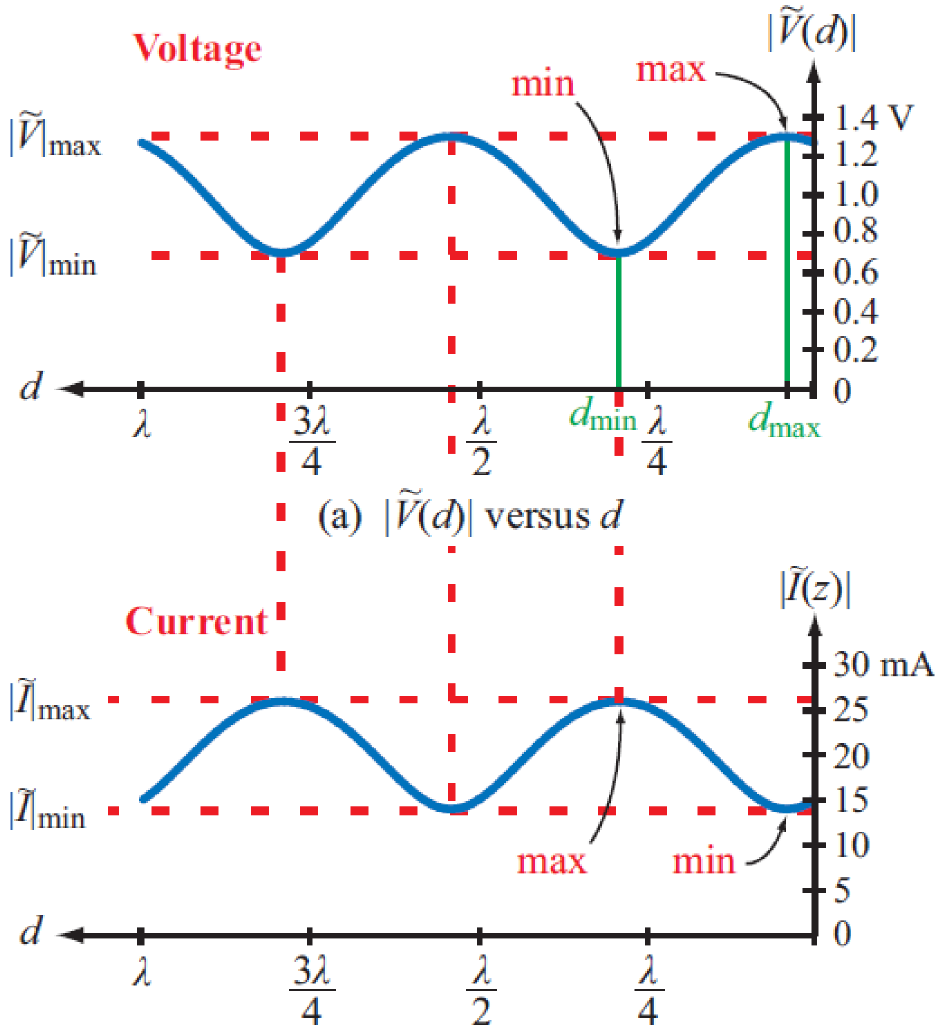
$$\text{when } (2\beta d_{\min} - \theta_r) = (2n+1)\pi$$

$$S = \frac{|\widetilde{V}|_{\max}}{|\widetilde{V}|_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad \text{(dimensionless)}$$

S = Voltage Standing Wave Ratio

For a matched load: S = 1

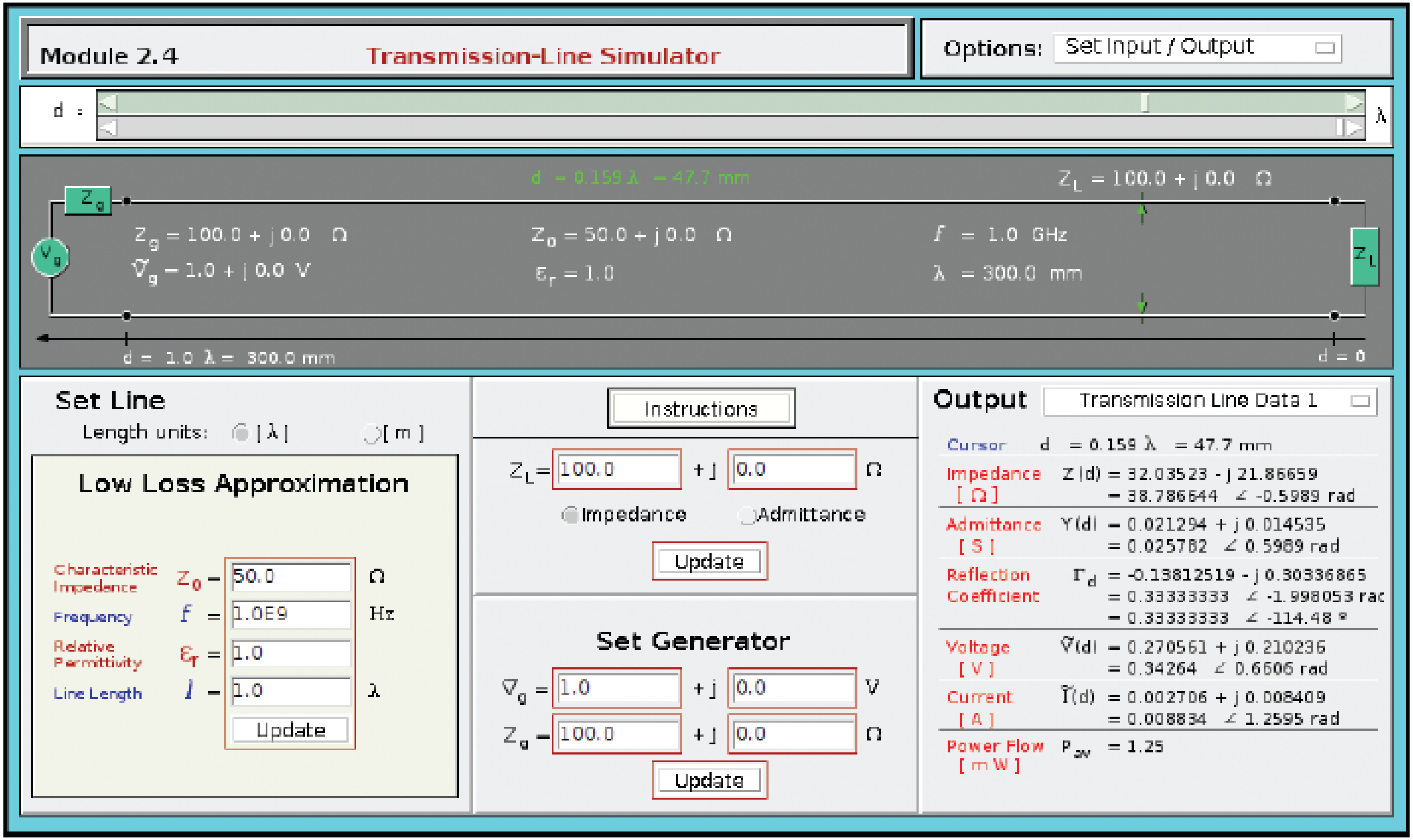
For a short, open, or purely reactive load: $S = \infty$.



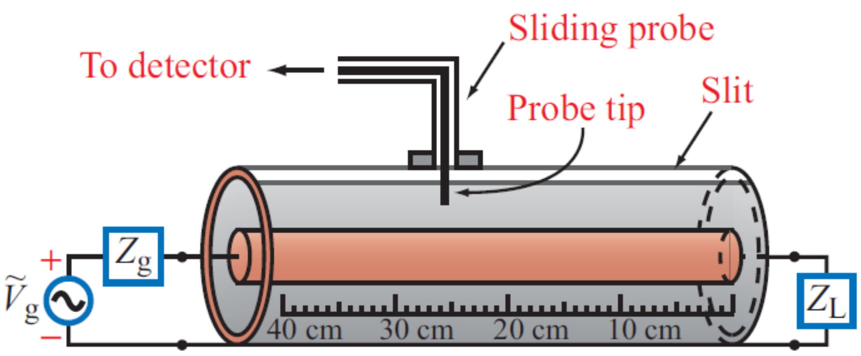
(a) $|\widetilde{V}(d)|$ versus d

(b) $|\widetilde{I}(d)|$ versus d

Module 2.4 Transmission-Line Simulator Upon specifying the requisite input data—including the load impedance at $d = 0$ and the generator voltage and impedance at $d = l$—this module provides a wealth of output information about the voltage and current waveforms along the trasmission line. You can view plots of the standing wave patterns for voltage and current, the time and spatial variations of the instantaneous voltage $\upsilon(d, t)$ and current $i(d, t)$, and other related quantities.



Example 2-6: Measuring Z_L with a Slotted Line



Solution: The following quantities are given:

$$Z_0 = 50\ \Omega,$$
$$S = 3,$$
$$d_{\min} = 12\text{ cm}.$$

Since the distance between successive voltage minima is $\lambda/2$,

$$\lambda = 2 \times 0.3 = 0.6\text{ m},$$

and

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{0.6} = \frac{10\pi}{3} \qquad \text{(rad/m)}.$$

From Eq. (2.73), solving for $|\Gamma|$ in terms of S gives

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = 0.5.$$

Next, we use the condition given by Eq. (2.71) to find θ_r:

$$2\beta d_{\min} - \theta_r = \pi, \qquad \text{for } n = 0 \text{ (first minimum)},$$

which gives

$$\theta_r = 2\beta d_{\min} - \pi = 2 \times \frac{10\pi}{3} \times 0.12 - \pi = -0.2\pi \text{ (rad)} = -36^\circ.$$

Hence,

$$\Gamma = |\Gamma| e^{j\theta_r} = 0.5 e^{-j36^\circ} = 0.405 - j0.294.$$

Solving Eq. (2.59) for Z_L, we have

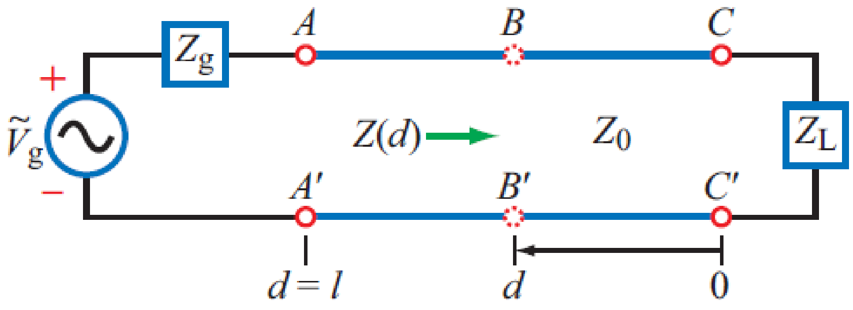
$$Z_L = Z_0 \left[\frac{1+\Gamma}{1-\Gamma}\right] = 50 \left[\frac{1 + 0.405 - j0.294}{1 - 0.405 + j0.294}\right]$$

$$\boxed{= (85 - j67)\ \Omega.}$$

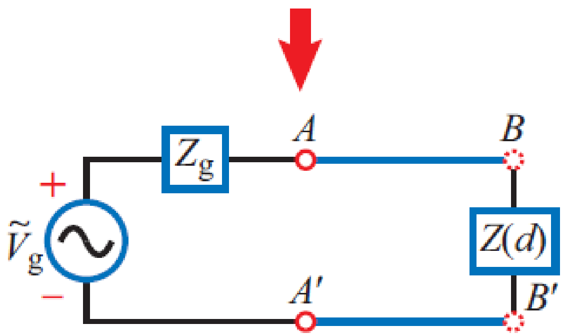
Wave Impedance

At a distance *d* from the load:

$$\begin{aligned} Z(d) &= \frac{\tilde{V}(d)}{\tilde{I}(d)} \\ &= \frac{V_0^+[e^{j\beta d} + \Gamma e^{-j\beta d}]}{V_0^+[e^{j\beta d} - \Gamma e^{-j\beta d}]} Z_0 \\ &= Z_0 \left[\frac{1+\Gamma e^{-j2\beta d}}{1-\Gamma e^{-j2\beta d}}\right] \\ &= Z_0 \left[\frac{1+\Gamma_d}{1-\Gamma_d}\right] \qquad (\Omega), \end{aligned}$$



(a) Actual circuit



(b) Equivalent circuit

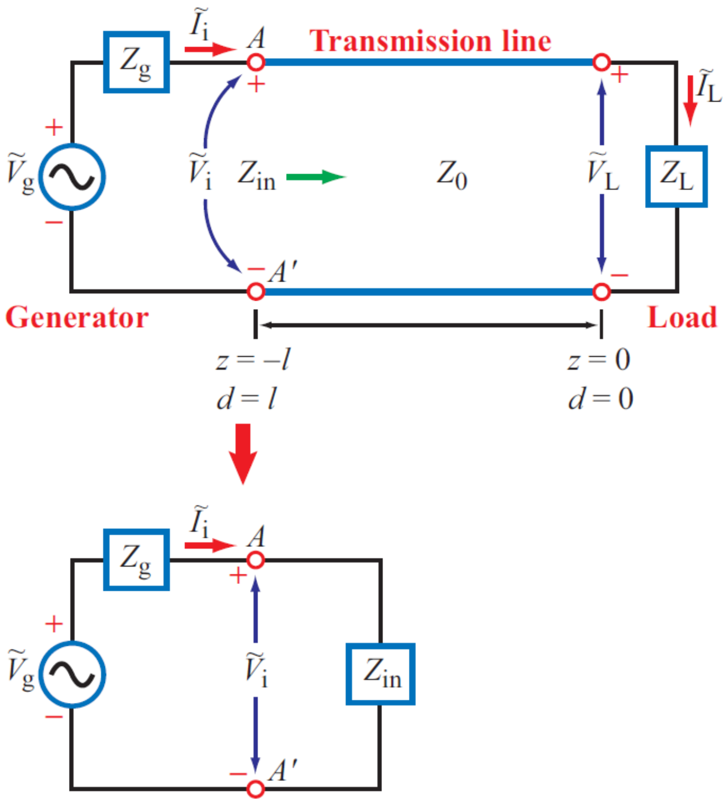
where we define

$$\Gamma_d = \Gamma e^{-j2\beta d} = |\Gamma| e^{j\theta_r} e^{-j2\beta d} = |\Gamma| e^{j(\theta_r - 2\beta d)}$$

as the ***phase-shifted voltage reflection coefficient***.

$Z(d)$ is the ratio of the total voltage (incident- and reflected-wave voltages) to the total current at any point d on the line, in contrast with the characteristic impedance of the line Z_0, which relates the voltage and current of each of the two waves individually ($Z_0 = V_0^+/I_0^+ = -V_0^-/I_0^-$).

Input Impedance



$$Z_{in} = Z_0 \left(\frac{z_L \cos\beta l + j \sin\beta l}{\cos\beta l + j z_L \sin\beta l} \right)$$
$$= Z_0 \left(\frac{z_L + j \tan\beta l}{1 + j z_L \tan\beta l} \right). \qquad (2.79)$$

Fig. 2-18. The phasor voltage across Z_{in} is given by

$$\tilde{V}_i = \tilde{I}_i Z_{in} = \frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}}, \qquad (2.80)$$

Simultaneously, from the standpoint of the transmission line, the voltage across it at the input of the line is given by Eq. (2.63a) with $z = -l$:

$$\tilde{V}_i = \tilde{V}(-l) = V_0^+[e^{j\beta l} + \Gamma e^{-j\beta l}]. \qquad (2.81)$$

Equating Eq. (2.80) to Eq. (2.81) and then solving for V_0^+ leads to

$$V_0^+ = \left(\frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}} \right) \left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}} \right). \qquad (2.82)$$

At input, $d = l$: $$Z_{in} = Z(l) = Z_0 \left[\frac{1 + \Gamma_l}{1 - \Gamma_l} \right].$$

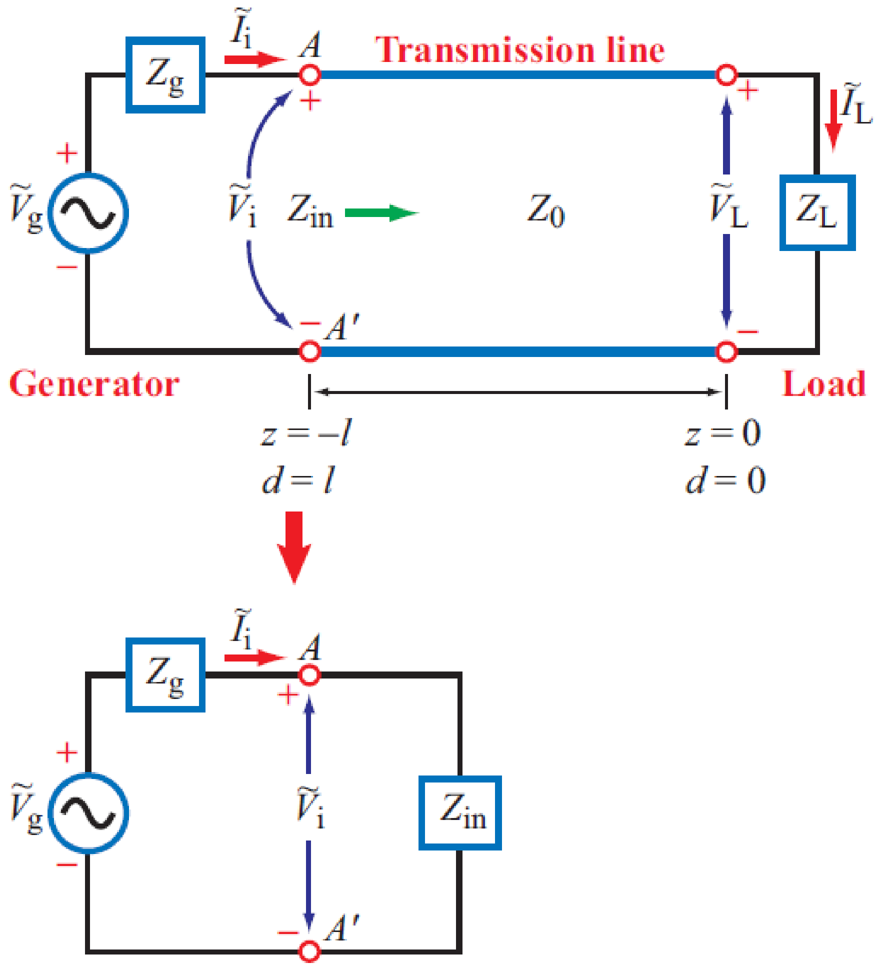
$$\Gamma_l = \Gamma e^{-j2\beta l} = |\Gamma| e^{j(\theta_r - 2\beta l)}.$$

Example 2-7: Complete Solution for $\upsilon(z,t)$ and $i(z,t)$

A 1.05-GHz generator circuit with series impedance $Z_g = 10\ \Omega$ and voltage source given by

$$\upsilon_g(t) = 10\sin(\omega t + 30^\circ) \qquad \text{(V)}$$

is connected to a load $Z_L = (100 + j50)\ \Omega$ through a 50-Ω, 67-cm long lossless transmission line. The phase velocity of the line is $0.7c$, where c is the velocity of light in a vacuum. Find $\upsilon(z,t)$ and $i(z,t)$ on the line.



Solution: From the relationship $u_p = \lambda f$, we find the wavelength

$$\lambda = \frac{u_p}{f} = \frac{0.7 \times 3 \times 10^8}{1.05 \times 10^9} = 0.2 \text{ m},$$

and

$$\beta l = \frac{2\pi}{\lambda}\, l = \frac{2\pi}{0.2} \times 0.67 = 6.7\pi = 0.7\pi = 126^\circ,$$

where we have subtracted multiples of 2π. The voltage reflection coefficient at the load is

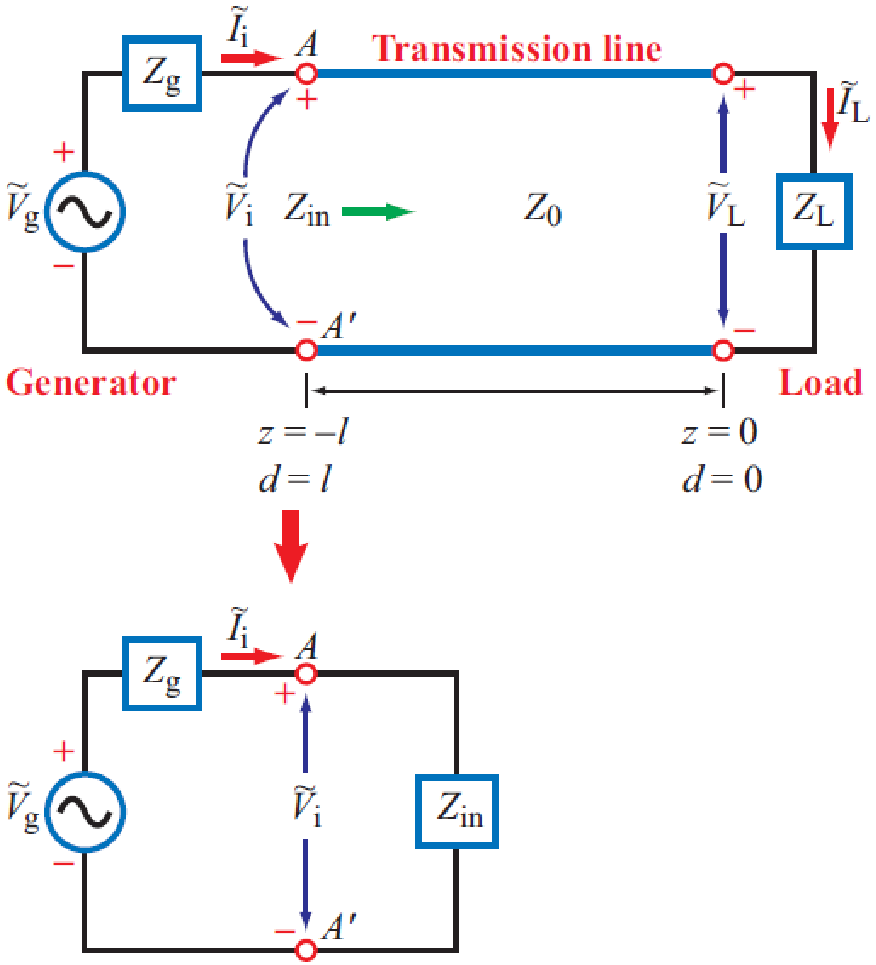
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(100 + j50) - 50}{(100 + j50) + 50} = 0.45e^{j26.6^\circ}.$$

Cont.

Example 2-7: Complete Solution for $v(z, t)$ and $i(z, t)$ (cont.)

A 1.05-GHz generator circuit with series impedance $Z_g = 10\ \Omega$ and voltage source given by

$$v_g(t) = 10 \sin(\omega t + 30°) \qquad \text{(V)}$$



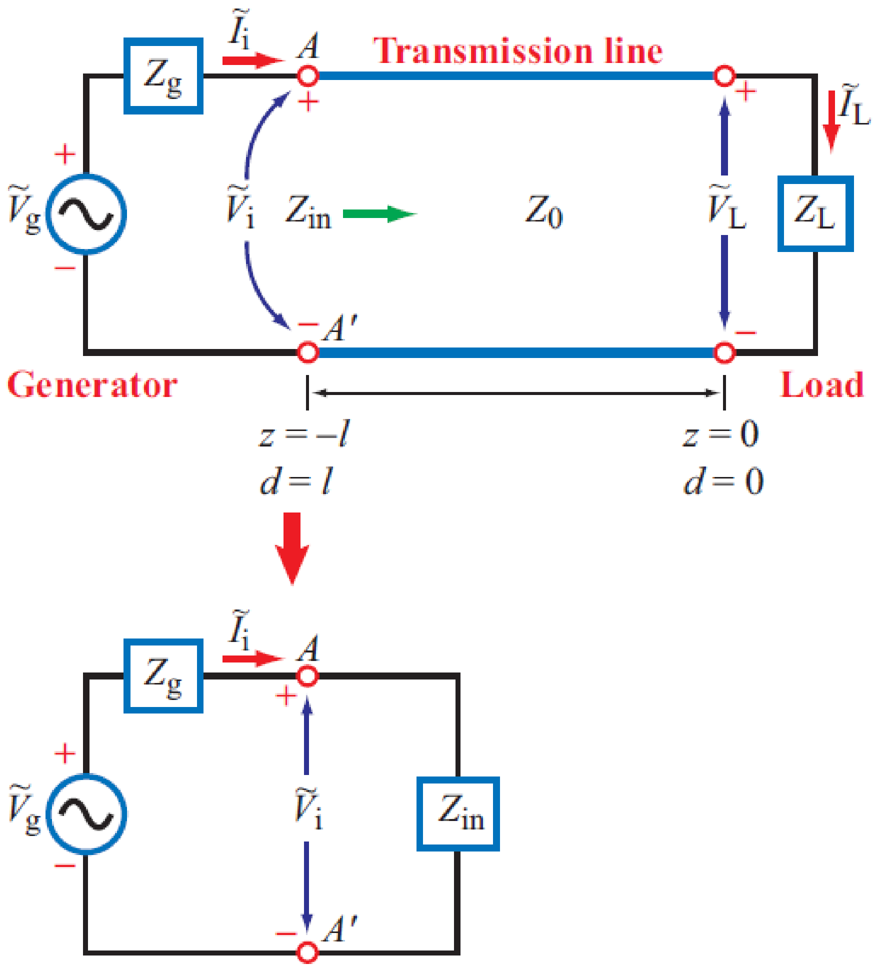
$$\begin{aligned}
Z_{in} &= Z_0 \left(\frac{1 + \Gamma_l}{1 - \Gamma_l} \right) \\
&= Z_0 \left(\frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} \right) \\
&= 50 \left(\frac{1 + 0.45 e^{j26.6°} e^{-j252°}}{1 - 0.45 e^{j26.6°} e^{-j252°}} \right) = (21.9 + j17.4)\ \Omega.
\end{aligned}$$

Rewriting the expression for the generator voltage with the cosine reference, we have

$$\begin{aligned}
v_g(t) &= 10 \sin(\omega t + 30°) \\
&= 10 \cos(90° - \omega t - 30°) \\
&= 10 \cos(\omega t - 60°) \\
&= \mathfrak{Re}[10 e^{-j60°} e^{j\omega t}] = \mathfrak{Re}[\tilde{V}_g e^{j\omega t}] \qquad \text{(V)}.
\end{aligned}$$

Cont.

Example 2-7: Complete Solution for $\upsilon(z, t)$ and $i(z, t)$ (cont.)



Hence, the phasor voltage $\tilde{V}_g$ is given by

$$\begin{aligned}\tilde{V}_g &= 10\,e^{-j60^\circ} \\ &= 10\angle{-60^\circ} \qquad \text{(V)}.\end{aligned}$$

Application of Eq. (2.82) gives

$$\begin{aligned}V_0^+ &= \left(\frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right) \\ &= \left[\frac{10e^{-j60^\circ}(21.9 + j17.4)}{10 + 21.9 + j17.4}\right] \\ &\quad \cdot (e^{j126^\circ} + 0.45e^{j26.6^\circ}e^{-j126^\circ})^{-1} \\ &= 10.2e^{j159^\circ} \qquad \text{(V)}.\end{aligned}$$

Using Eq. (2.63a) with $z = -d$, the phasor voltage on the line is

$$\begin{aligned}\tilde{V}(d) &= V_0^+(e^{j\beta d} + \Gamma e^{-j\beta d}) \\ &= 10.2e^{j159^\circ}(e^{j\beta d} + 0.45e^{j26.6^\circ}e^{-j\beta d}),\end{aligned}$$

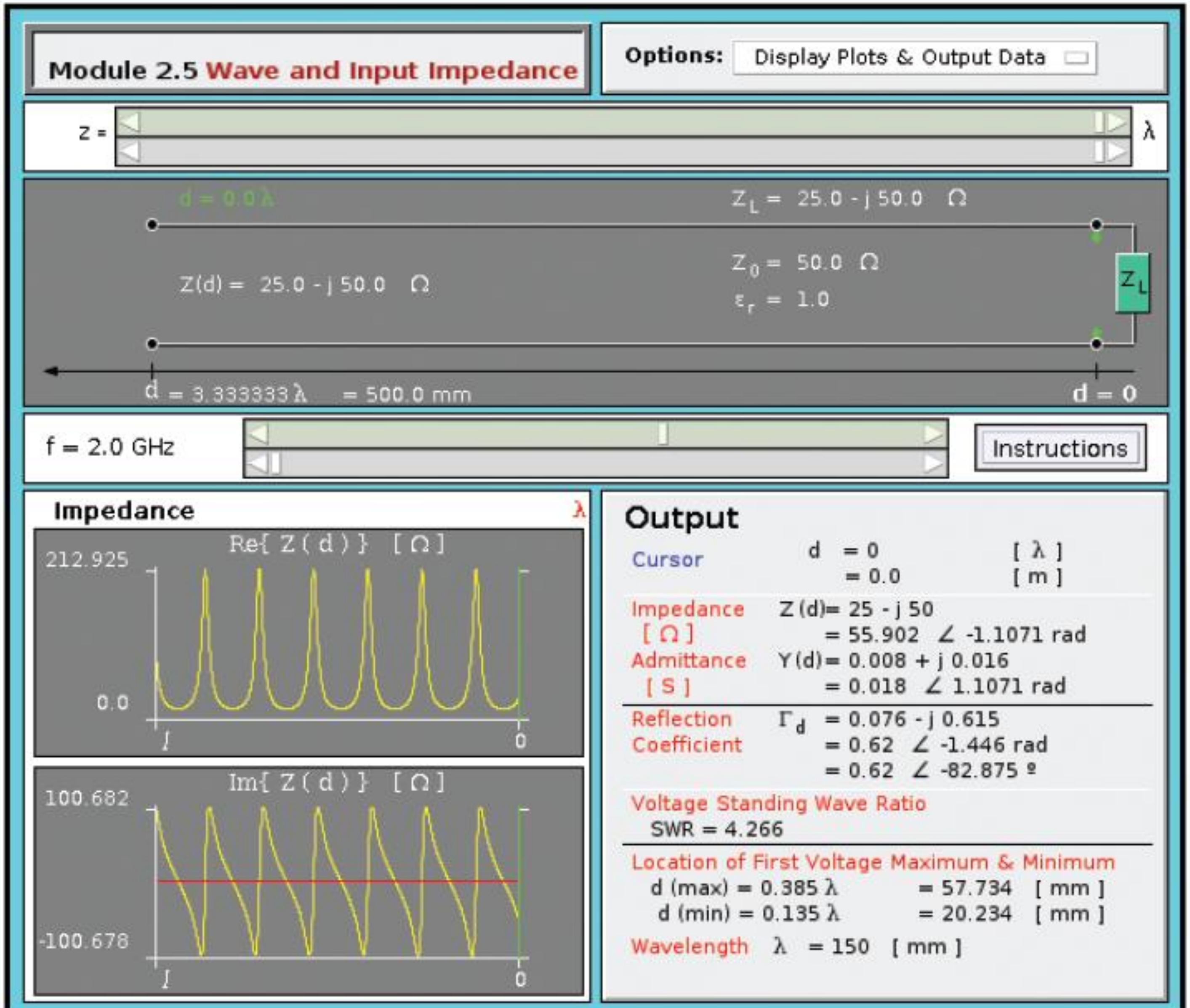
and the corresponding instantaneous voltage $\upsilon(d, t)$ is

$$\boxed{\begin{aligned}\upsilon(d, t) &= \mathfrak{Re}[\tilde{V}(d)\,e^{j\omega t}] \\ &= 10.2\cos(\omega t + \beta d + 159^\circ) \\ &\quad + 4.55\cos(\omega t - \beta d + 185.6^\circ) \qquad \text{(V)}.\end{aligned}}$$

Similarly, Eq. (2.63b) leads to

$$\boxed{\begin{aligned}\tilde{I}(d) &= 0.20e^{j159^\circ}(e^{j\beta d} - 0.45e^{j26.6^\circ}e^{-j\beta d}), \\ i(d, t) &= 0.20\cos(\omega t + \beta d + 159^\circ) \\ &\quad + 0.091\cos(\omega t - \beta d + 185.6^\circ) \qquad \text{(A)}.\end{aligned}}$$

Module 2.5 Wave and Input Impedance The wave impedance, $Z(d) = \tilde{V}(d)/\tilde{I}(d)$, exhibits a cyclical pattern as a function of position along the line. This module displays plots of the real and imaginary parts of $Z(d)$, specifies the locations of the voltage maximum and minimum nearest to the load, and provides other related information.



Short-Circuited Line

For the short-circuited line: $\Gamma = -1$

$$\widetilde{V}_{sc}(d) = V_0^+[e^{j\beta d} - e^{-j\beta d}] = 2jV_0^+ \sin\beta d,$$

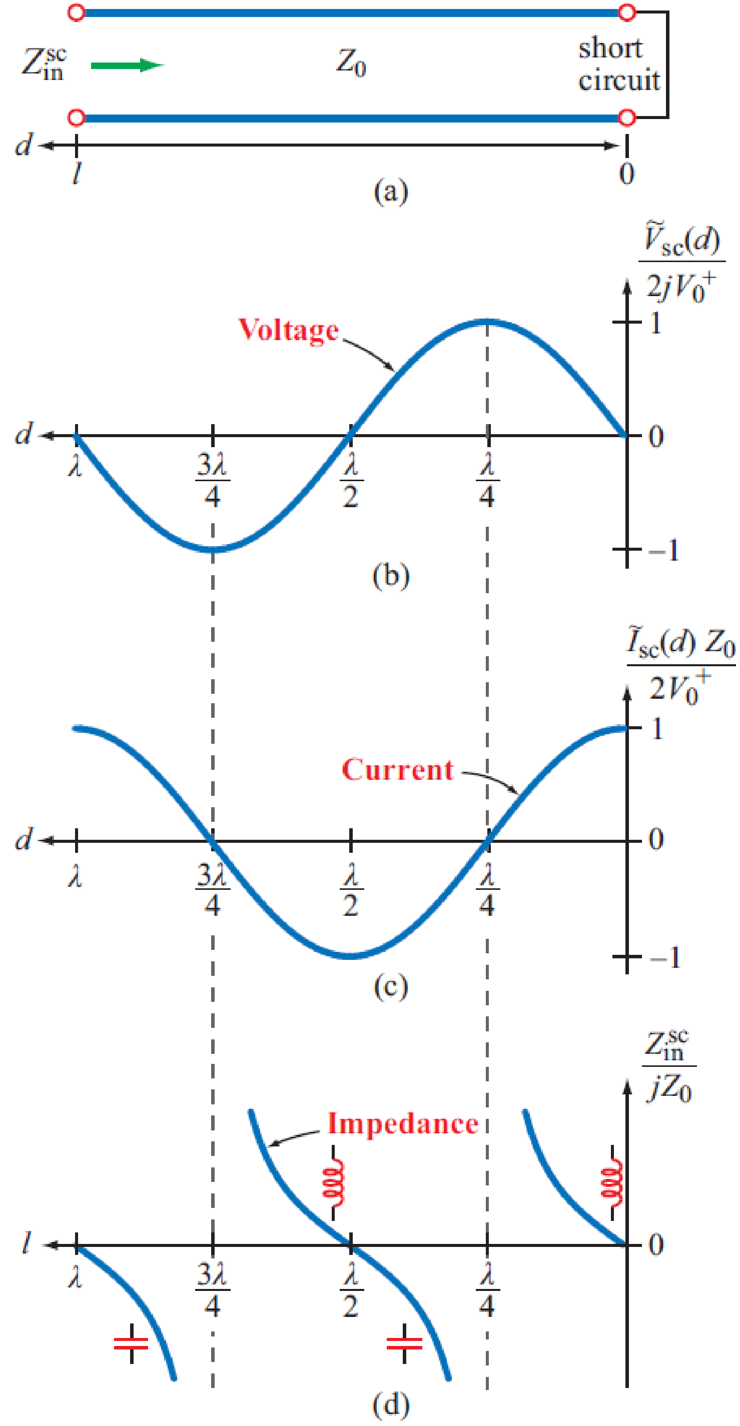
$$\widetilde{I}_{sc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} + e^{-j\beta d}] = \frac{2V_0^+}{Z_0}\cos\beta d,$$

$$Z_{sc}(d) = \frac{\widetilde{V}_{sc}(d)}{\widetilde{I}_{sc}(d)} = jZ_0 \tan\beta d.$$

At its input, the line appears like an inductor or a capacitor depending on the sign of $\tan\beta d$

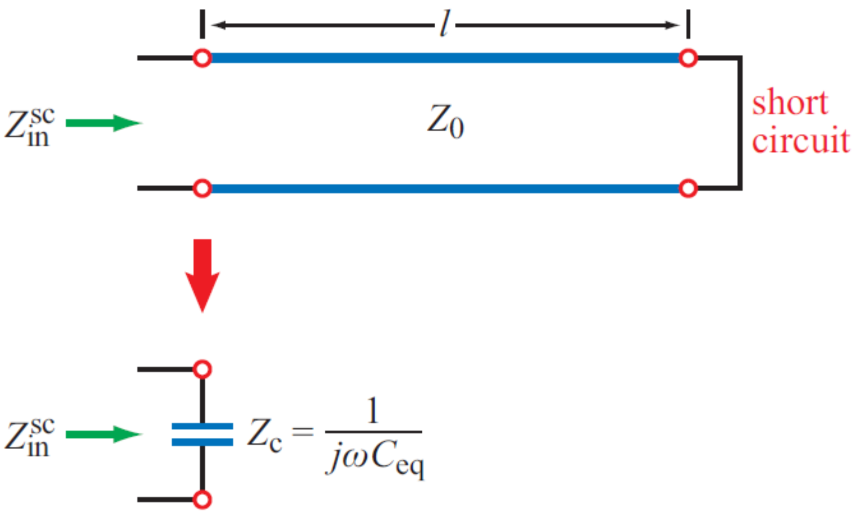
$$j\omega L_{eq} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \ge 0$$

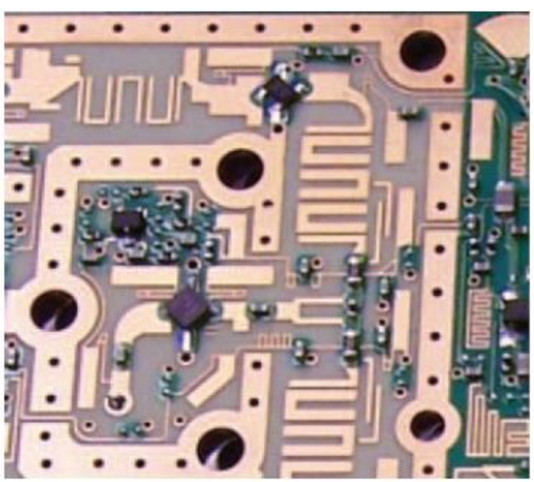
$$\frac{1}{j\omega C_{eq}} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \le 0$$



Example 2-8: Equivalent Reactive Elements

Choose the length of a shorted 50-Ω lossless transmission line (Fig. 2-20) such that its input impedance at 2.25 GHz is identical to that of a capacitor with capacitance $C_{\text{eq}} = 4$ pF. The wave velocity on the line is $0.75c$.





Solution: We are given

$$u_{\text{p}} = 0.75c = 0.75 \times 3 \times 10^8 = 2.25 \times 10^8 \text{ m/s},$$

$$Z_0 = 50\ \Omega,$$

$$f = 2.25 \text{ GHz} = 2.25 \times 10^9 \text{ Hz},$$

$$C_{\text{eq}} = 4 \text{ pF} = 4 \times 10^{-12} \text{ F}.$$

The phase constant is

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{u_{\text{p}}} = \frac{2\pi \times 2.25 \times 10^9}{2.25 \times 10^8} = 62.8 \qquad \text{(rad/m)}.$$

From Eq. (2.89), it follows that

$$\begin{aligned} \tan \beta l &= -\frac{1}{Z_0 \omega C_{\text{eq}}} \\ &= -\frac{1}{50 \times 2\pi \times 2.25 \times 10^9 \times 4 \times 10^{-12}} \\ &= -0.354. \end{aligned}$$

The tangent function is negative when its argument is in the second or fourth quadrants. The solution for the second quadrant is

$$\beta l_1 = 2.8 \text{ rad} \quad \text{or} \quad \boxed{l_1 = \frac{2.8}{\beta} = \frac{2.8}{62.8} = 4.46 \text{ cm},}$$

and the solution for the fourth quadrant is

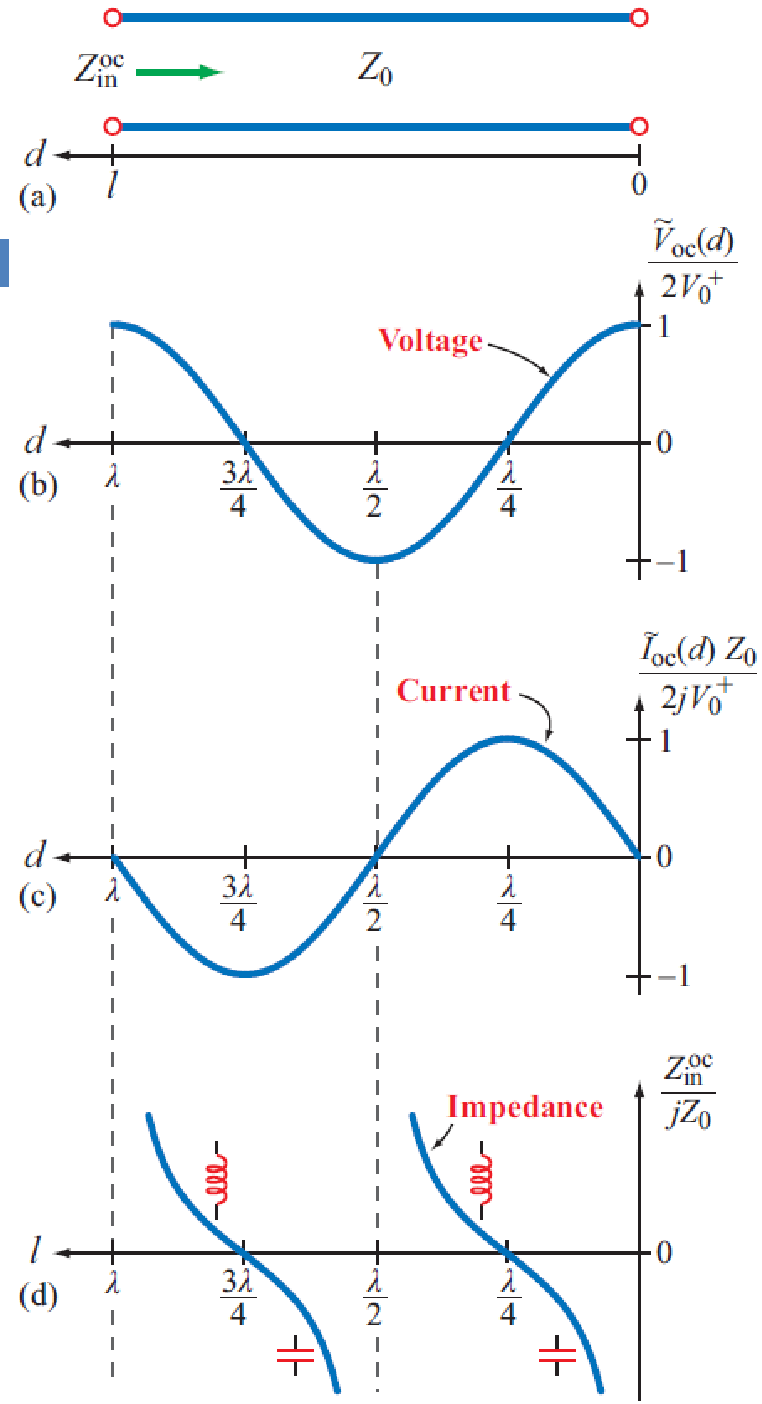
$$\beta l_2 = 5.94 \text{ rad} \quad \text{or} \quad \boxed{l_2 = \frac{5.94}{62.8} = 9.46 \text{ cm}.}$$

Open-Circuted Line

$$\tilde{V}_{oc}(d) = V_0^+[e^{j\beta d} + e^{-j\beta d}] = 2V_0^+ \cos\beta d,$$

$$\tilde{I}_{oc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} - e^{-j\beta d}] = \frac{2jV_0^+}{Z_0}\sin\beta d,$$

$$\boxed{Z_{in}^{oc} = \frac{\tilde{V}_{oc}(l)}{\tilde{I}_{oc}(l)} = -jZ_0 \cot\beta l. \qquad (2.93)}$$



Short-Circuit/Open-Circuit Method

- For a line of known length *l*, measurements of its input impedance, one when terminated in a short and another when terminated in an open, can be used to find its characteristic impedance Z_0 and electrical length βl

$$Z_{\text{in}}^{\text{sc}} = \frac{\widetilde{V}_{\text{sc}}(l)}{\widetilde{I}_{\text{sc}}(l)} = jZ_0 \tan \beta l.$$

$$Z_{\text{in}}^{\text{oc}} = \frac{\widetilde{V}_{\text{oc}}(l)}{\widetilde{I}_{\text{oc}}(l)} = -jZ_0 \cot \beta l.$$

$$Z_0 = +\sqrt{Z_{\text{in}}^{\text{sc}}\, Z_{\text{in}}^{\text{oc}}}\,,$$

$$\tan \beta l = \sqrt{\frac{-Z_{\text{in}}^{\text{sc}}}{Z_{\text{in}}^{\text{oc}}}}\,.$$

2-8.4 Lines of Length $l = n\lambda/2$

If $l = n\lambda/2$, where n is an integer,

$$\tan\beta l = \tan[(2\pi/\lambda)(n\lambda/2)] = \tan n\pi = 0.$$

Consequently, Eq. (2.79) reduces to

$$\boxed{Z_{\text{in}} = Z_{\text{L}}, \qquad \text{for } l = n\lambda/2, \qquad (2.96)}$$

which means that a half-wavelength line (or any integer multiple of $\lambda/2$) does not modify the load impedance.

2-8.5 Quarter-Wavelength Transformer

$$Z_{\text{in}} = \frac{Z_0^2}{Z_{\text{L}}}, \qquad \text{for } l = \lambda/4 + n\lambda/2.$$

Example 2-10: $\lambda/4$ Transformer

A 50-Ω lossless transmission line is to be matched to a resistive load impedance with $Z_{\text{L}} = 100$ Ω via a quarter-wave section as shown in Fig. 2-22, thereby eliminating reflections along the feedline. Find the required characteristic impedance of the quarter-wave transformer.

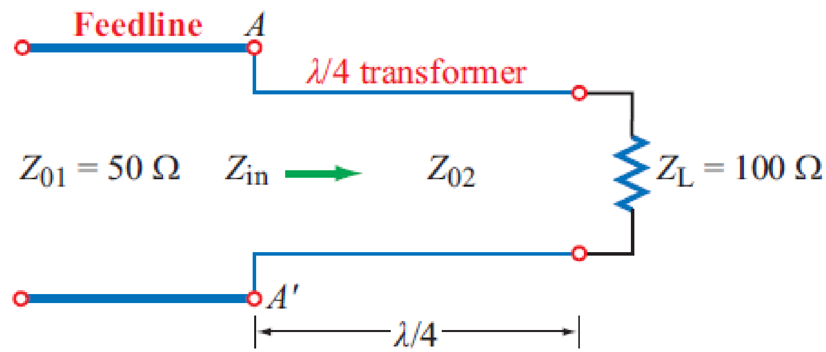


Figure 2-22: Configuration for Example 2-10.

Solution: To eliminate reflections at terminal AA', the input impedance Z_{in} looking into the quarter-wave line should be equal to Z_{01}, the characteristic impedance of the feedline. Thus, $Z_{\text{in}} = 50$ Ω. From Eq. (2.97),

$$Z_{\text{in}} = \frac{Z_{02}^2}{Z_{\text{L}}},$$

or

$$Z_{02} = \sqrt{Z_{\text{in}} Z_{\text{L}}} = \sqrt{50 \times 100} = 70.7\ \Omega.$$

Whereas this eliminates reflections on the feedline, it does not eliminate them on the $\lambda/4$ line.

Table 2-4: Properties of standing waves on a lossless transmission line.

Voltage Maximum	$\lvert\widetilde{V}\rvert_{\max} = \lvert V_0^+\rvert[1+\lvert\Gamma\rvert]$
Voltage Minimum	$\lvert\widetilde{V}\rvert_{\min} = \lvert V_0^+\rvert[1-\lvert\Gamma\rvert]$
Positions of voltage maxima (also positions of current minima)	$d_{\max} = \dfrac{\theta_r\lambda}{4\pi} + \dfrac{n\lambda}{2}, \quad n = 0, 1, 2, \ldots$
Position of first maximum (also position of first current minimum)	$d_{\max} = \begin{cases} \dfrac{\theta_r\lambda}{4\pi}, & \text{if } 0 \le \theta_r \le \pi \\ \dfrac{\theta_r\lambda}{4\pi} + \dfrac{\lambda}{2}, & \text{if } -\pi \le \theta_r \le 0 \end{cases}$
Positions of voltage minima (also positions of current maxima)	$d_{\min} = \dfrac{\theta_r\lambda}{4\pi} + \dfrac{(2n+1)\lambda}{4}, \quad n = 0, 1, 2, \ldots$
Position of first minimum (also position of first current maximum)	$d_{\min} = \dfrac{\lambda}{4}\left(1 + \dfrac{\theta_r}{\pi}\right)$
Input Impedance	$Z_{\text{in}} = Z_0\left(\dfrac{z_L + j\tan\beta l}{1 + jz_L\tan\beta l}\right) = Z_0\left(\dfrac{1+\Gamma_l}{1-\Gamma_l}\right)$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{\text{in}} = Z_0\left(\dfrac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}\right)$
Z_{in} at voltage minima	$Z_{\text{in}} = Z_0\left(\dfrac{1-\lvert\Gamma\rvert}{1+\lvert\Gamma\rvert}\right)$
Z_{in} of short-circuited line	$Z_{\text{in}}^{\text{sc}} = jZ_0\tan\beta l$
Z_{in} of open-circuited line	$Z_{\text{in}}^{\text{oc}} = -jZ_0\cot\beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{\text{in}} = Z_L, \quad n = 0, 1, 2, \ldots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$	$Z_{\text{in}} = Z_0^2/Z_L, \quad n = 0, 1, 2, \ldots$
Z_{in} of matched line	$Z_{\text{in}} = Z_0$

$\lvert V_0^+\rvert$ = amplitude of incident wave; $\Gamma = \lvert\Gamma\rvert e^{j\theta_r}$ with $-\pi < \theta_r < \pi$; θ_r in radians; $\Gamma_l = \Gamma e^{-j2\beta l}$.

Instantaneous Power Flow

$$\begin{aligned} v(d,t) &= \mathfrak{Re}[\widetilde{V} e^{j\omega t}] \\ &= \mathfrak{Re}[|V_0^+| e^{j\phi^+}(e^{j\beta d} + |\Gamma| e^{j\theta_r} e^{-j\beta d}) e^{j\omega t}] \\ &= |V_0^+|[\cos(\omega t + \beta d + \phi^+) \\ &\qquad + |\Gamma| \cos(\omega t - \beta d + \phi^+ + \theta_r)], \end{aligned} \qquad (2.99a)$$

$$\begin{aligned} i(d,t) &= \frac{|V_0^+|}{Z_0}[\cos(\omega t + \beta d + \phi^+) \\ &\qquad - |\Gamma| \cos(\omega t - \beta d + \phi^+ + \theta_r)], \end{aligned} \qquad (2.99b)$$

$$\begin{aligned} P(d,t) &= v(d,t)\, i(d,t) \\ &= |V_0^+|[\cos(\omega t + \beta d + \phi^+) \\ &\qquad + |\Gamma| \cos(\omega t - \beta d + \phi^+ + \theta_r)] \\ &\quad \times \frac{|V_0^+|}{Z_0}[\cos(\omega t + \beta d + \phi^+) \\ &\qquad - |\Gamma| \cos(\omega t - \beta d + \phi^+ + \theta_r)] \\ &= \frac{|V_0^+|^2}{Z_0}[\cos^2(\omega t + \beta d + \phi^+) \\ &\qquad - |\Gamma|^2 \cos^2(\omega t - \beta d + \phi^+ + \theta_r)] \end{aligned}$$

$$P^{\mathrm{i}}(d,t) = \frac{|V_0^+|^2}{Z_0} \cos^2(\omega t + \beta d + \phi^+) \qquad \text{(W)},$$

$$P^{\mathrm{r}}(d,t) = -|\Gamma|^2 \frac{|V_0^+|^2}{Z_0} \cos^2(\omega t - \beta d + \phi^+ + \theta_r)$$

Using the trigonometric identity

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x),$$

the expressions in Eq. (2.101) can be rewritten as

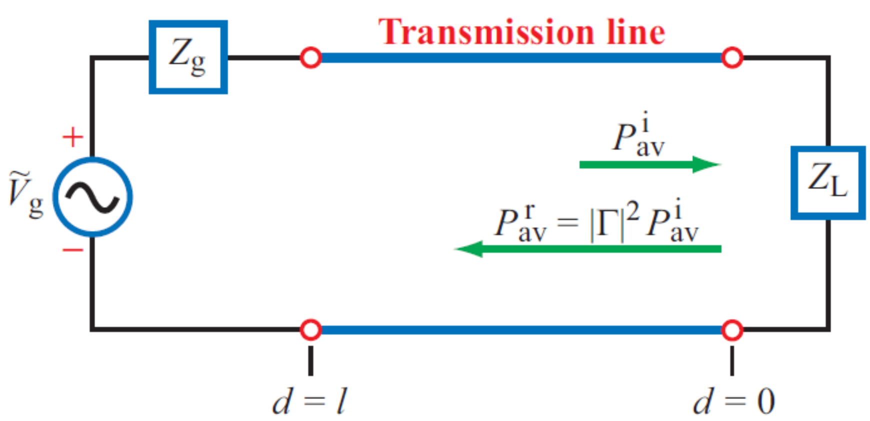
$$\boxed{\begin{aligned} P^{\mathrm{i}}(d,t) &= \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t + 2\beta d + 2\phi^+)], \\ P^{\mathrm{r}}(d,t) &= -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t - 2\beta d \\ &\qquad + 2\phi^+ + 2\theta_r)]. \end{aligned}}$$

The power oscillates at twice the rate of the voltage or current.

Average Power

$$P^{\mathrm{i}}(d,t) = \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t + 2\beta d + 2\phi^+)],$$

$$P^{\mathrm{r}}(d,t) = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t - 2\beta d + 2\phi^+ + 2\theta_{\mathrm{r}})].$$



$$P_{\mathrm{av}}^{\mathrm{i}}(d) = \frac{1}{T}\int_0^T P^{\mathrm{i}}(d,t)\,dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} P^{\mathrm{i}}(d,t)\,dt. \qquad (2.103)$$

Upon inserting Eq. (2.102a) into Eq. (2.103) and performing the integration, we obtain

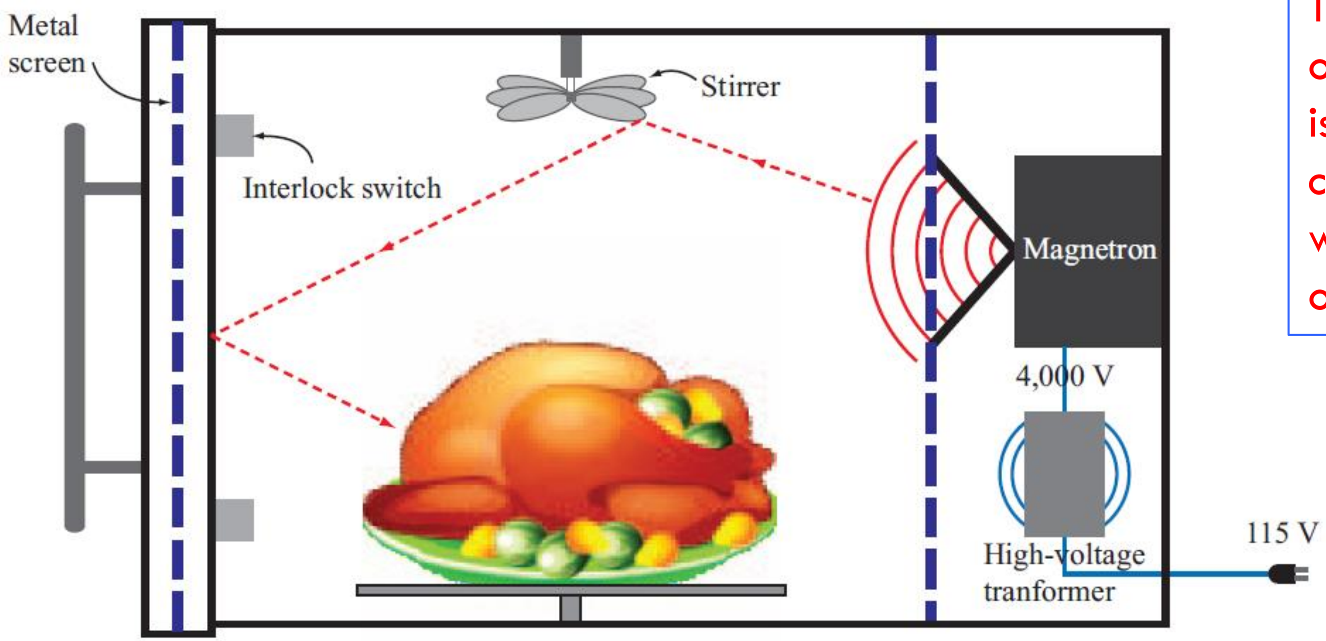
$$\boxed{P_{\mathrm{av}}^{\mathrm{i}} = \frac{|V_0^+|^2}{2Z_0} \quad (\mathrm{W}),} \qquad (2.104)$$

which is identical with the dc term of $P^{\mathrm{i}}(d,t)$ given by Eq. (2.102a). A similar treatment for the reflected wave gives

$$\boxed{P_{\mathrm{av}}^{\mathrm{r}} = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0} = -|\Gamma|^2 P_{\mathrm{av}}^{\mathrm{i}}.} \qquad (2.105)$$

The average reflected power is equal to the average incident power, diminished by a multiplicative factor of $|\Gamma|^2$.

Tech Brief 3: Standing Waves in Microwave Oven



(a) Cavity

The stirrer or rotation of the food platform is used to constantly change the standing wave pattern in the oven cavity

Tech Brief 3: Role of Frequency

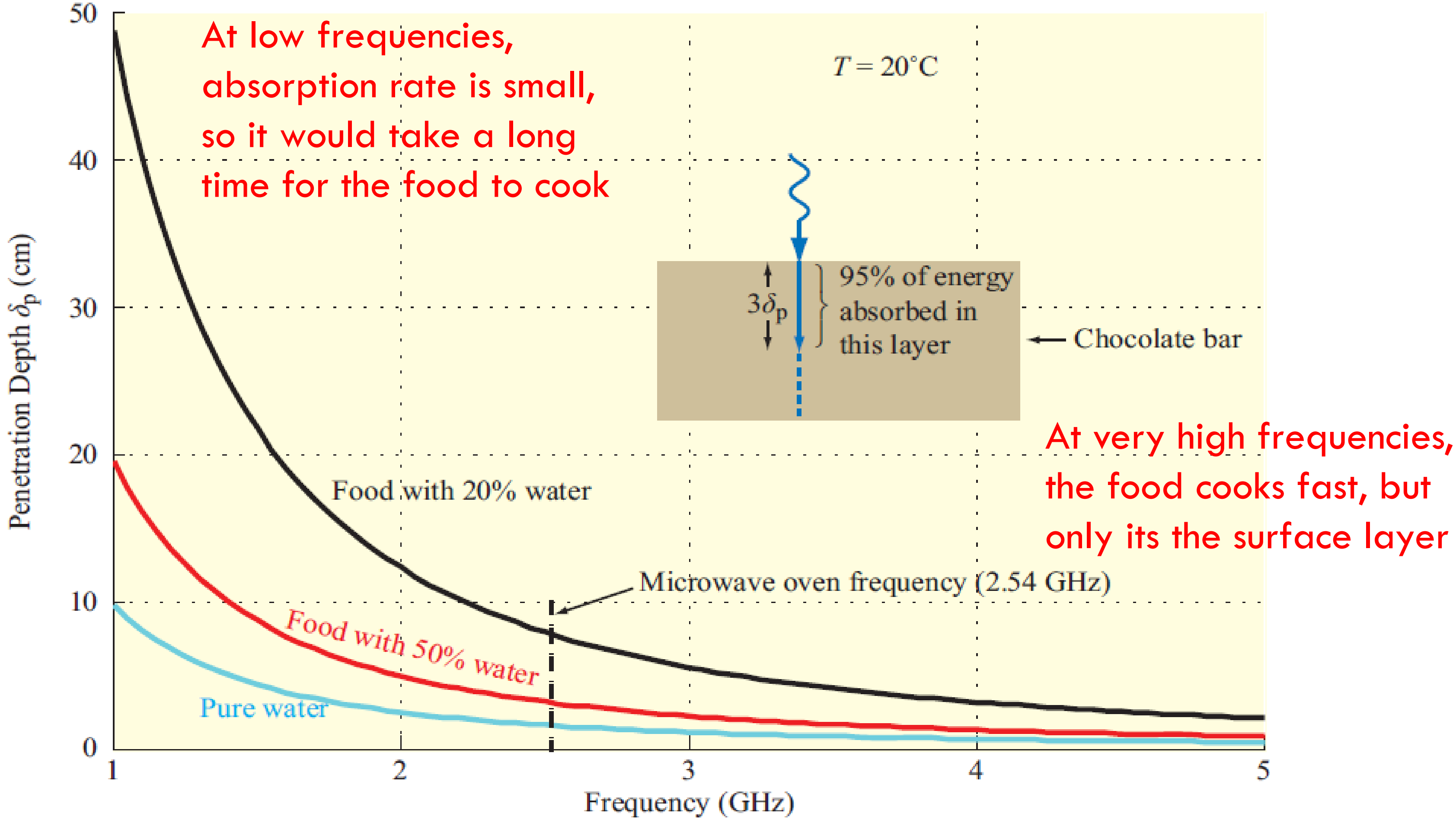
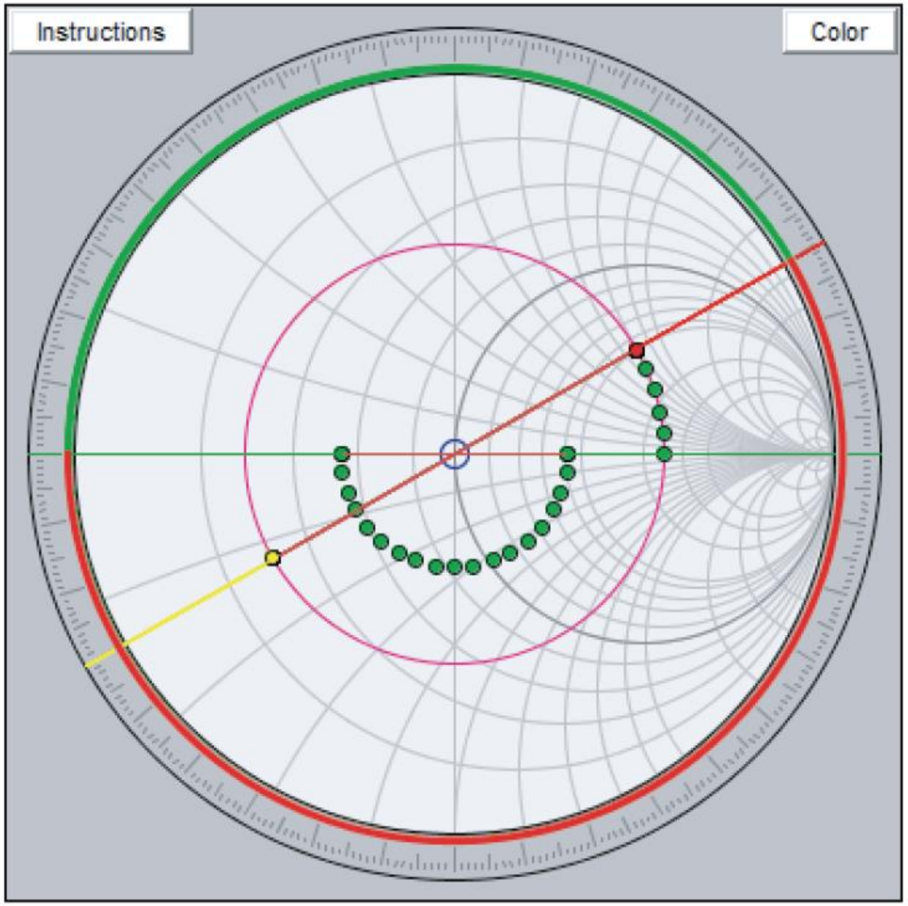


Figure TF3-1: Penetration depth as a function of frequency (1–5 GHz) for pure water and two foods with different water contents.

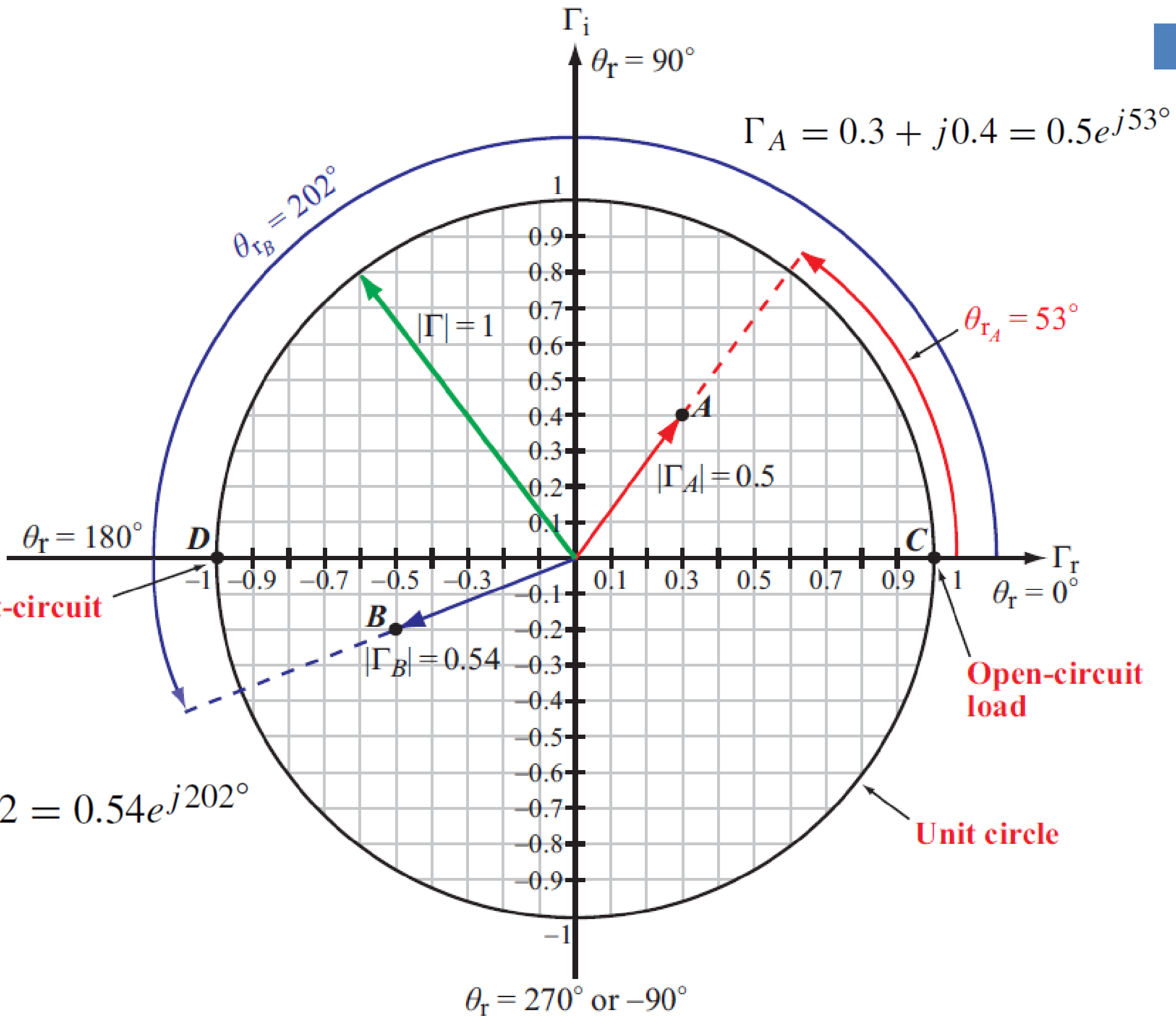
The Smith Chart

- Developed in 1939 by P. W. Smith as a graphical tool to analyze and design transmission-line circuits
- Today, it is used to characterize the performance of microwave circuits



Complex Plane

$$\Gamma = |\Gamma| e^{j\theta_r} = \Gamma_r + j\Gamma_i$$



$$\Gamma_B = -0.5 - j0.2 = 0.54e^{j202°}$$

Smith Chart Parametric Equations

$$\Gamma = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z_L - 1}{z_L + 1}$$

$$\boxed{z_L = \frac{1 + \Gamma}{1 - \Gamma}\,. \qquad (2.112)}$$

$$z_L = r_L + j x_L.$$

$$r_L + j x_L = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i}$$

$$r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

$$x_L = \frac{2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

Equation for a circle

$$\left(\Gamma_r - \frac{r_L}{1 + r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1 + r_L}\right)^2. \qquad (2.116)$$

The standard equation for a circle in the x–y plane with center at (x_0, y_0) and radius a is

$$(x - x_0)^2 + (y - y_0)^2 = a^2. \qquad (2.117)$$

A similar manipulation of the expression for x_L given by Eq. (2.115b) leads to

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2, \qquad (2.118)$$

Smith Chart Parametric Equations

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2. \qquad (2.116)$$

r_L circles

r_L circles are contained inside the unit circle

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2, \qquad (2.118)$$

x_L circles

Only parts of the x_L circles are contained within the unit circle

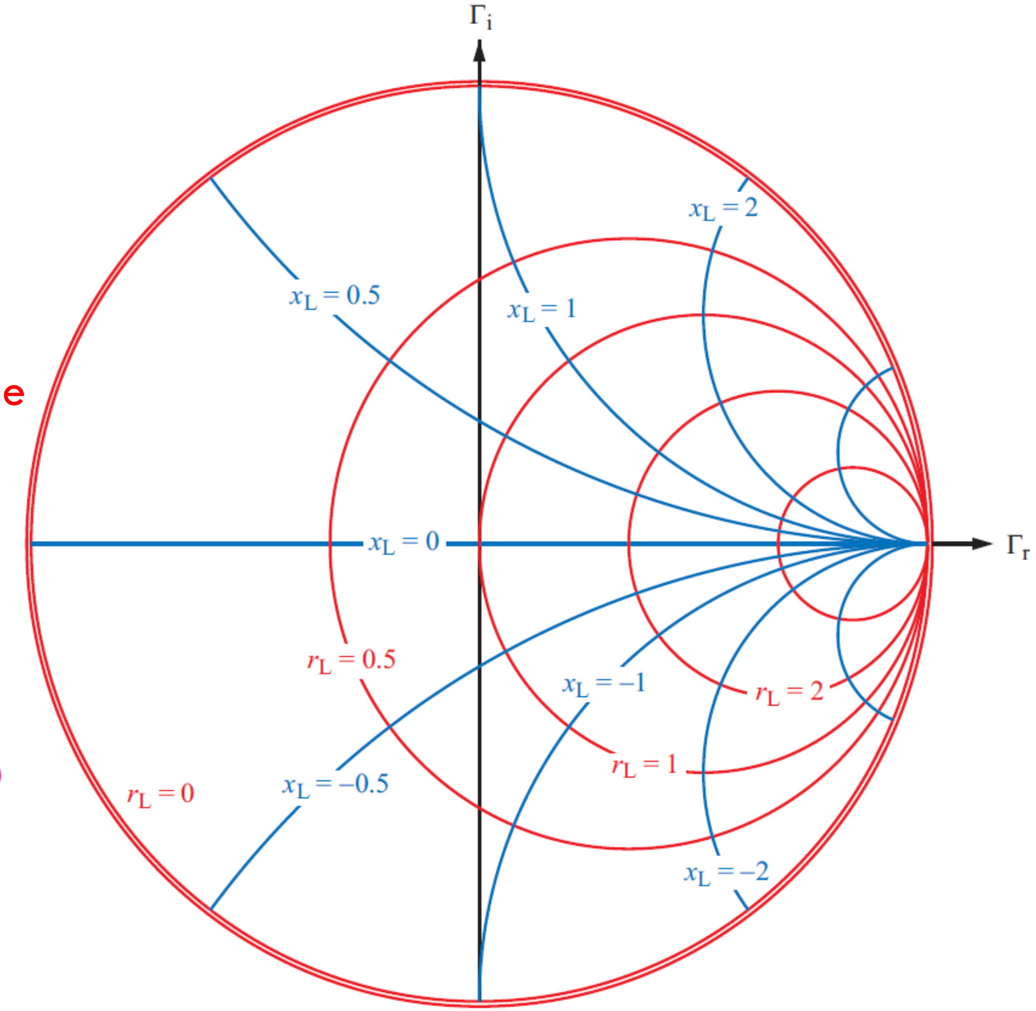
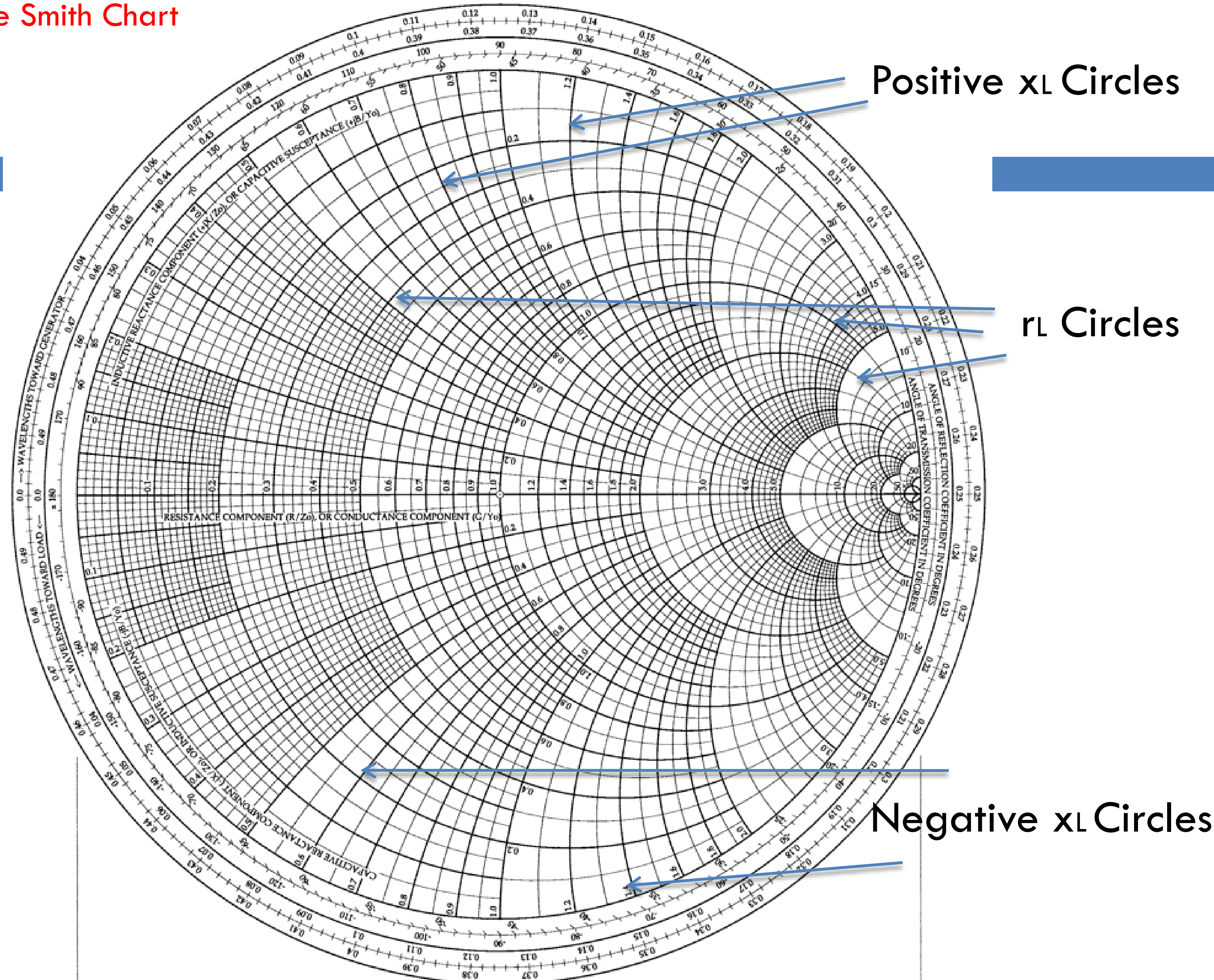


Figure 2-25: Families of r_L and x_L circles within the domain $|\Gamma| \leq 1$.

Complete Smith Chart



Reflection coefficient at the load

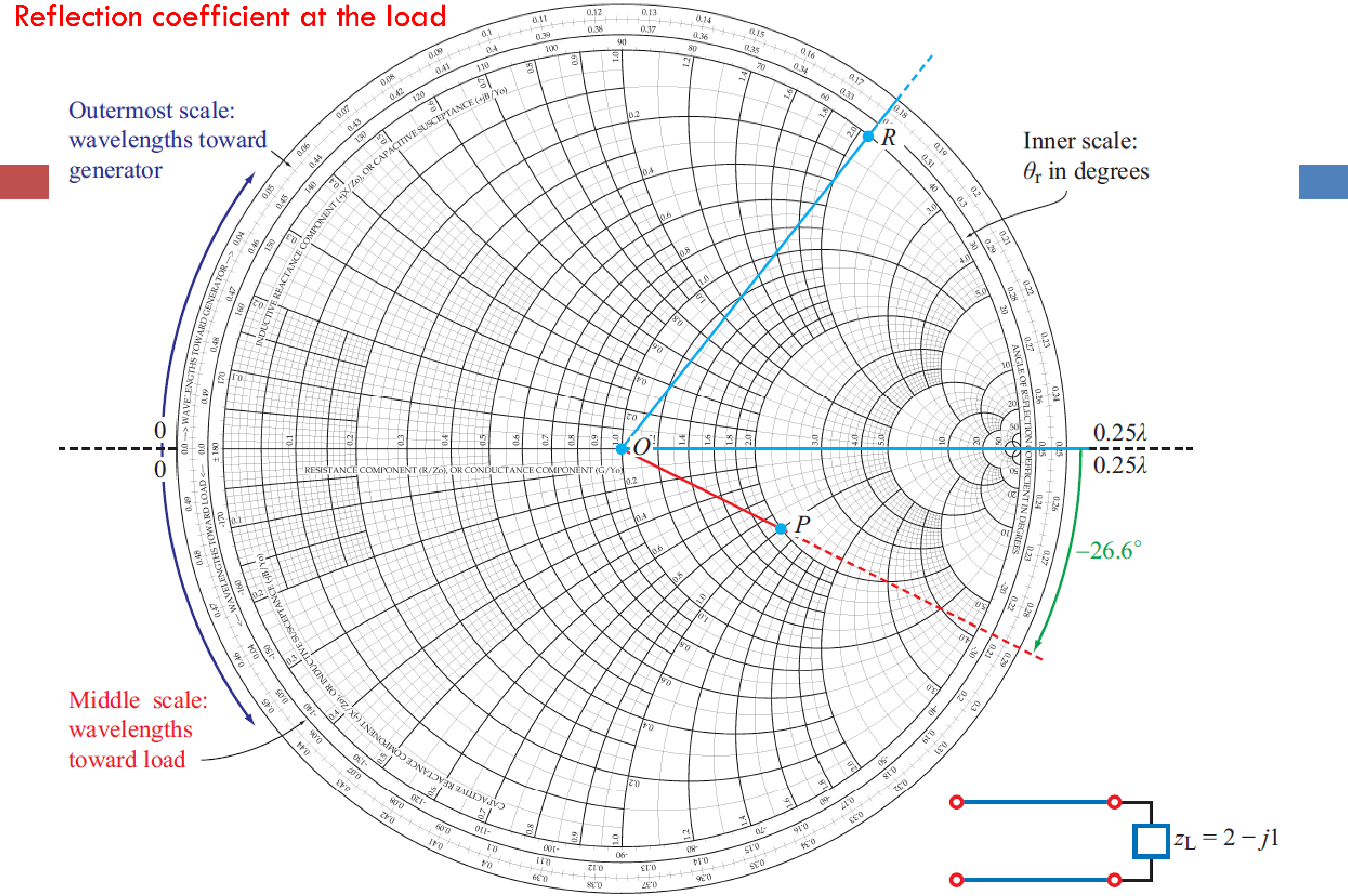


Figure 2-26: Point P represents a normalized load impedance $z_L = 2 - j1$. The reflection coefficient has a magnitude $|\Gamma| = \overline{OP}/\overline{OR} = 0.45$ and an angle $\theta_r = -26.6°$. Point R is an arbitrary point on the $r_L = 0$ circle (which also is the $|\Gamma| = 1$ circle).

Input Impedance

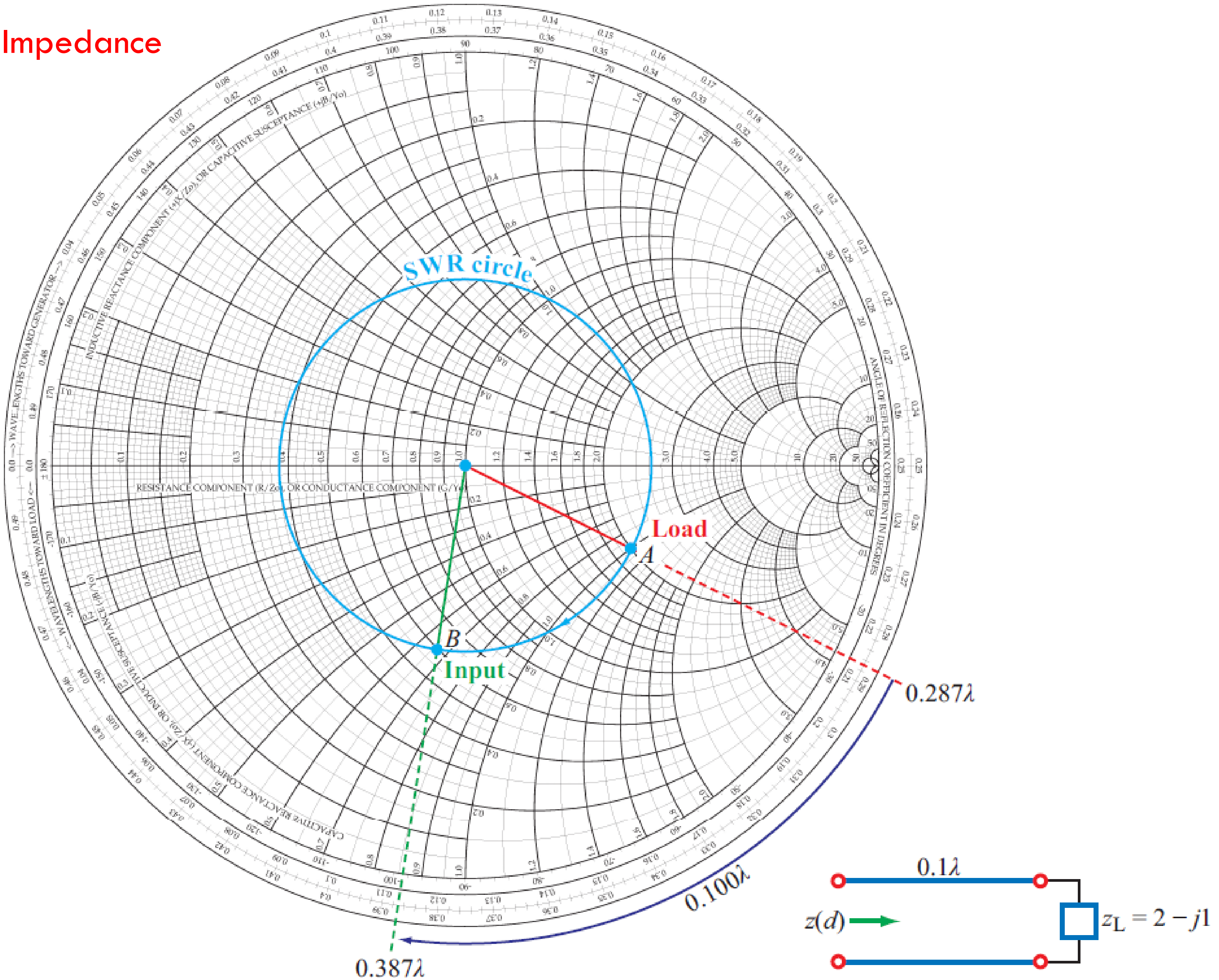


Figure 2-27: Point A represents a normalized load $z_L = 2 - j1$ at 0.287λ on the WTG scale. Point B represents the line input at $d = 0.1\lambda$ from the load. At B, $z(d) = 0.6 - j0.66$.

Maxima and Minima

S is numerically equal to the value of r_0 at P_{max}, the point at which the SWR circle intersects the real Γ axis to the right of the chart's center.

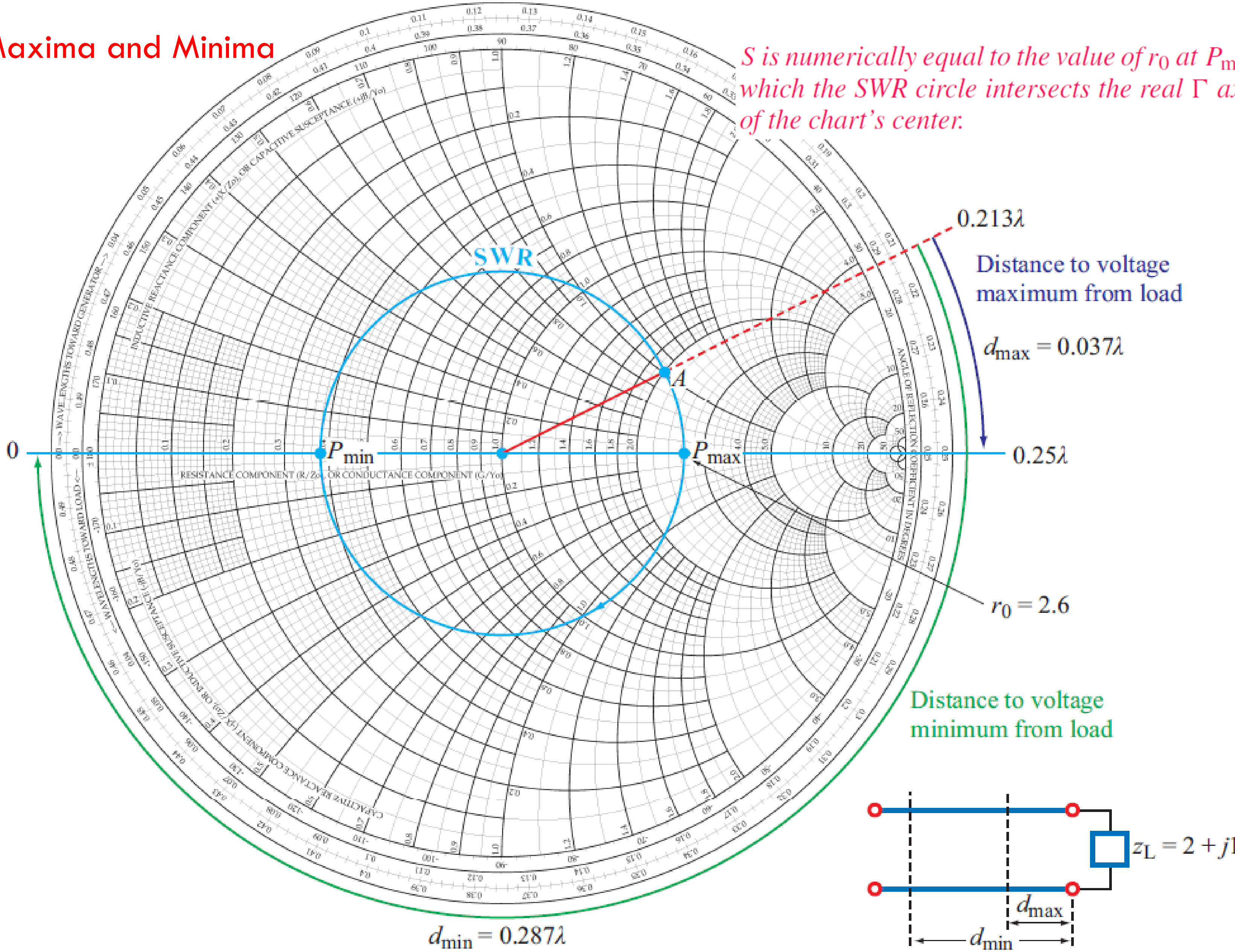
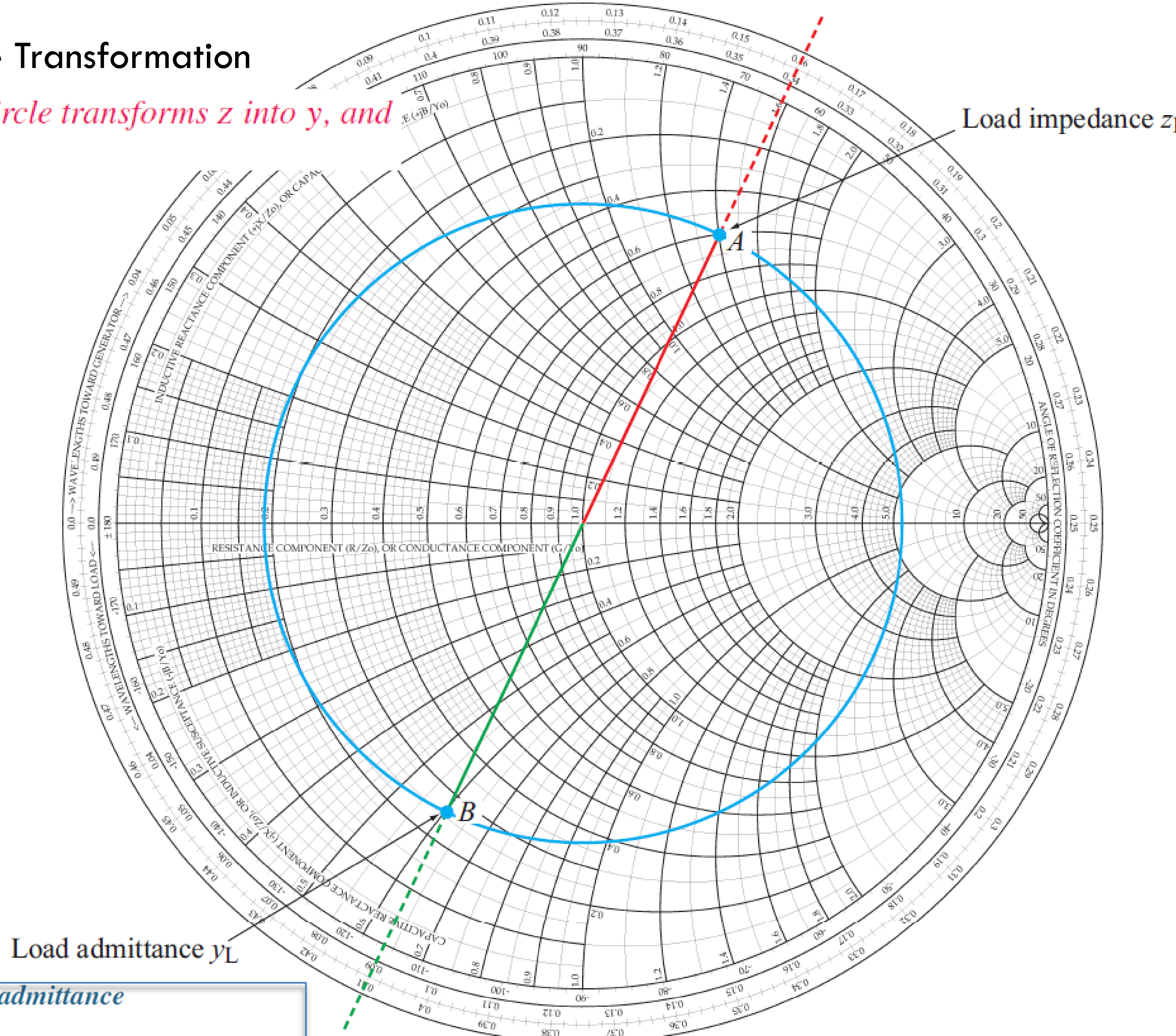


Figure 2-28: Point A represents a normalized load with $z_L = 2 + j1$. The standing wave ratio is $S = 2.6$ (at P_{max}), the distance between the load and the first voltage maximum is $d_{max} = (0.25 - 0.213)\lambda = 0.037\lambda$, and the distance between the load and the first voltage minimum is $d_{min} = (0.037 + 0.25)\lambda = 0.287\lambda$.

Impedance to Admittance Transformation

Rotation by λ/4 on the SWR circle transforms z into y, and vice versa.



normalized admittance

$$y = \frac{Y}{Y_0} = \frac{G}{Y_0} + j\frac{B}{Y_0} = g + jb \qquad \text{(dimensionless)}, \quad (2.132)$$

where $Y_0 = 1/Z_0$ is the ***characteristic admittance of the line***

Example 2-11: Smith Chart Calculations

A 50-Ω lossless transmission line of length 3.3λ is terminated by a load impedance $Z_L = (25 + j50)$ Ω.

(a)

$$z_L = \frac{Z_L}{Z_0} = \frac{25 + j50}{50} = 0.5 + j1$$

$$d_{max} = (0.25 - 0.135)\lambda = 0.115\lambda$$

$$d_{min} = (0.5 - 0.135)\lambda = 0.365\lambda$$

(b)

$$z_{in} = 0.28 - j0.40$$

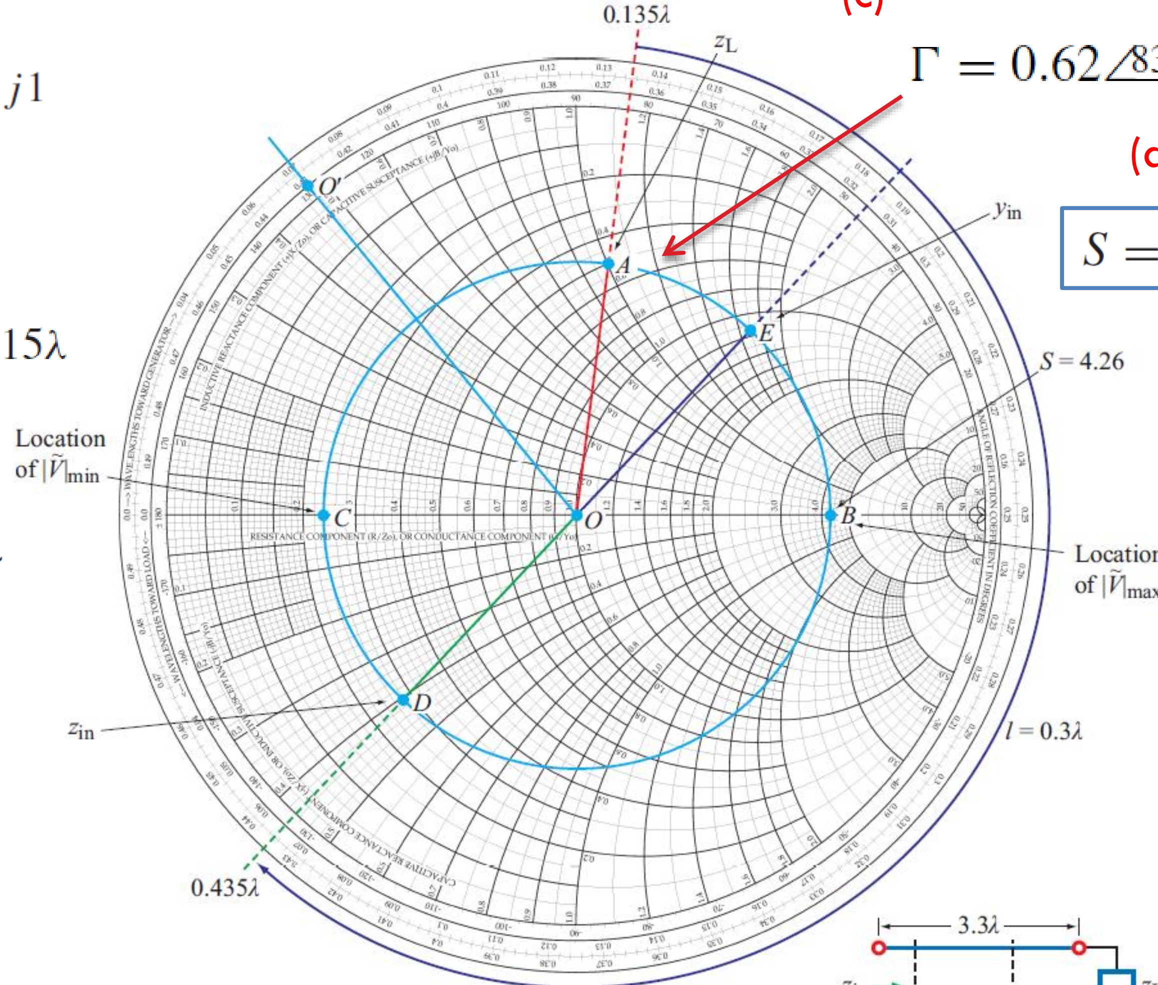
$$Z_{in} = z_{in} Z_0 = (0.28 - j0.40)50 = (14 - j20)\ \Omega$$

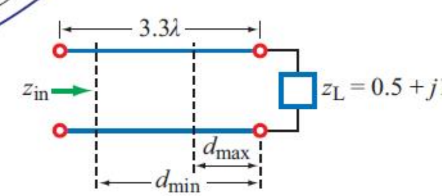
(c)

$$\Gamma = 0.62\angle 83^\circ$$

(d)

$$\boxed{S = 4.26}$$





Example 2-12: Determining Z_L Using the Smith Chart

Given:

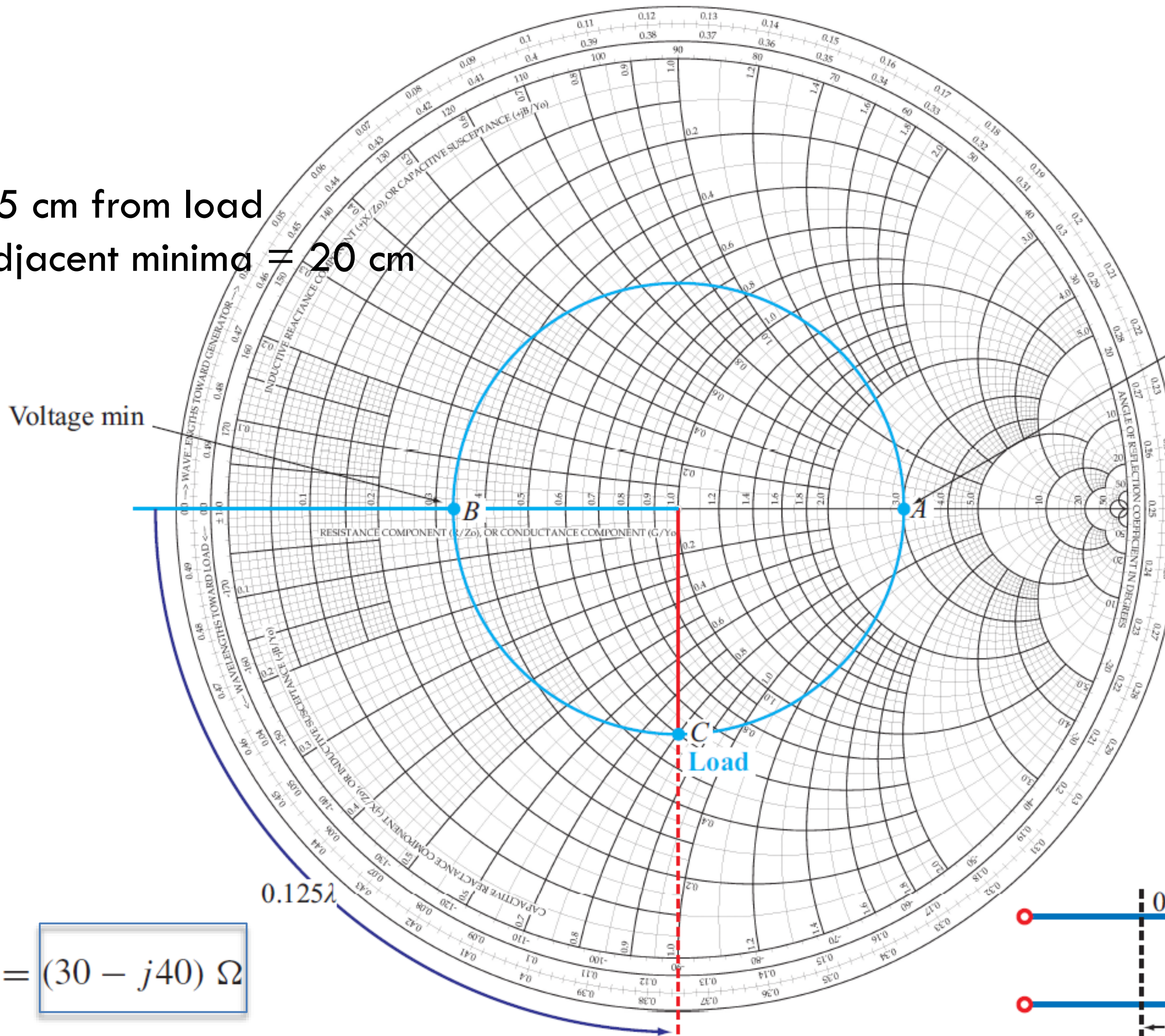
$S = 3$

$Z_0 = 50\ \Omega$

first voltage min @ 5 cm from load

Distance between adjacent minima = 20 cm

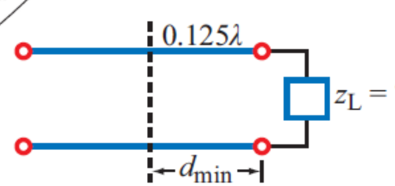
Determine: Z_L



$$d_{\min} = \frac{5}{40} = 0.125\lambda$$

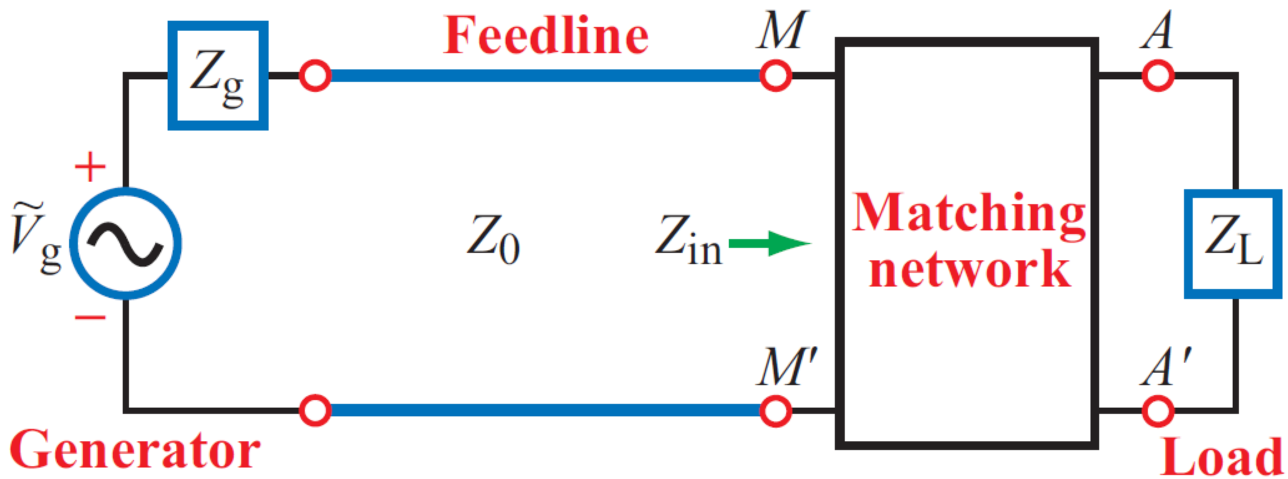
$$z_L = 0.6 - j0.8$$

$$Z_L = 50(0.6 - j0.8) = \boxed{(30 - j40)\ \Omega}$$

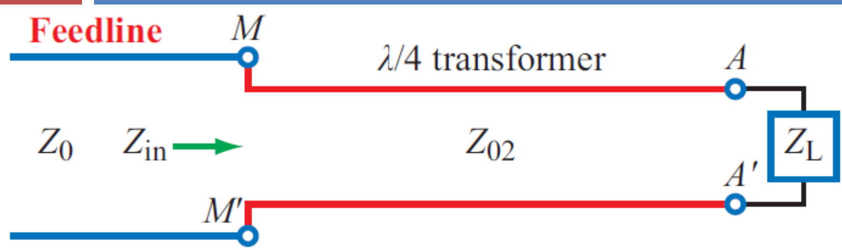


Matching Networks

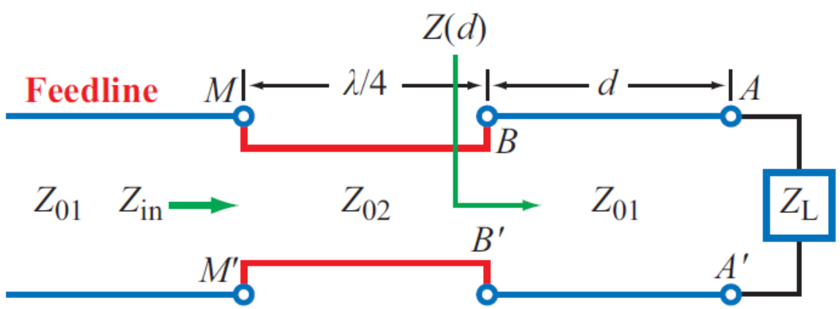
The purpose of the matching network is to eliminate reflections at terminals MM' for waves incident from the source. Even though multiple reflections may occur between AA' and MM', only a forward traveling wave exists on the feedline.



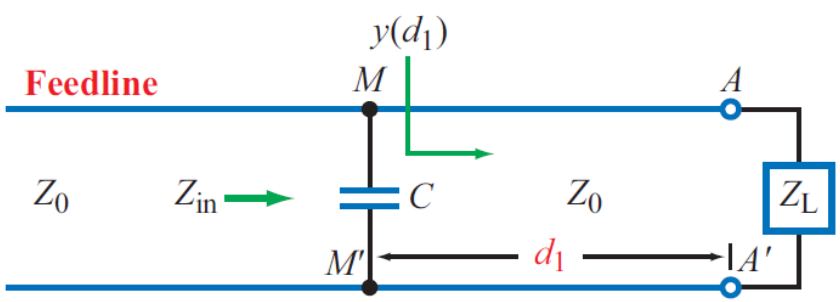
Examples of Matching Networks



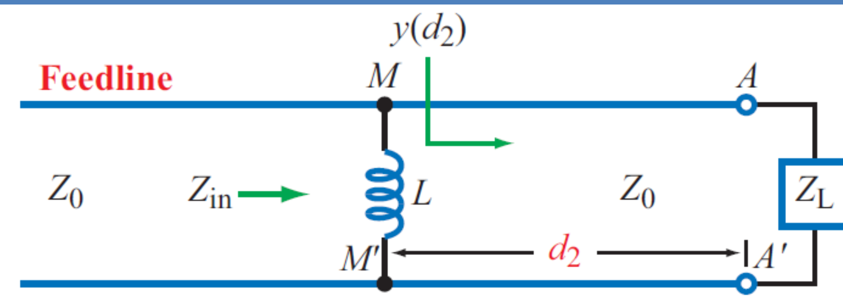
(a) In-series $\lambda/4$ transformer inserted at AA'



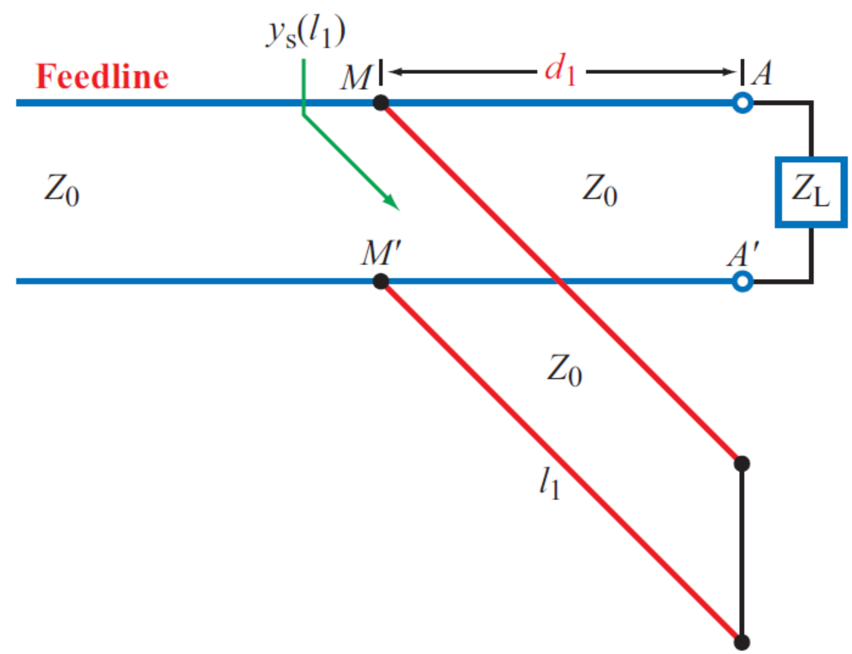
(b) In-series $\lambda/4$ transformer inserted at $d = d_{\max}$ or $d = d_{\min}$



(c) In-parallel insertion of capacitor at distance d_1



(d) In-parallel insertion of inductor at distance d_2



(e) In-parallel insertion of a short-circuited stub

Lumped-Element Matching

Choose d and Ys to achieve a match at MM'

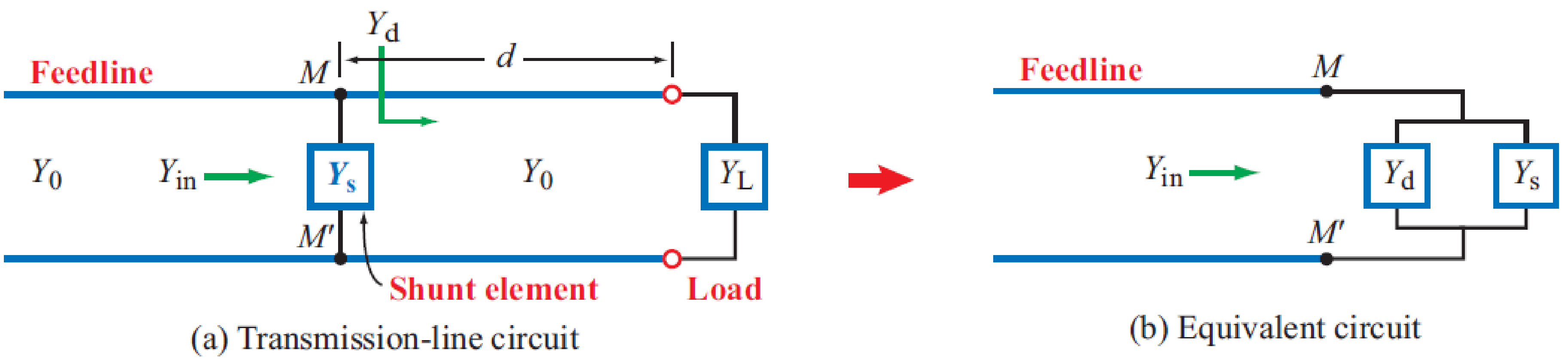


Figure 2-34: Inserting a reactive element with admittance Y_s at MM' modifies Y_d to Y_{in}.

$$y_{in} = g_d + j(b_d + b_s). \tag{2.140}$$

$$Y_{in} = Y_d + Y_s$$

$$Y_{in} = (G_d + jB_d) + jB_s$$
$$= G_d + j(B_d + B_s).$$

To achieve a matched condition at MM', it is necessary that $y_{in} = 1 + j0$, which translates into two specific conditions, namely

$$g_d = 1 \quad \text{(real-part condition)}, \tag{2.141a}$$

$$b_s = -b_d \quad \text{(imaginary-part condition)}. \tag{2.141b}$$

Example 2-13: Lumped Element

A load impedance $Z_L = 25 - j50\ \Omega$ is connected to a 50-Ω transmission line. Insert a shunt element to eliminate reflections towards the sending end of the line. Specify the insert location d (in wavelengths), the type of element, and its value, given that $f = 100$ MHz.

$$z_L = \frac{Z_L}{Z_0} = \frac{25 - j50}{50} = 0.5 - j1$$

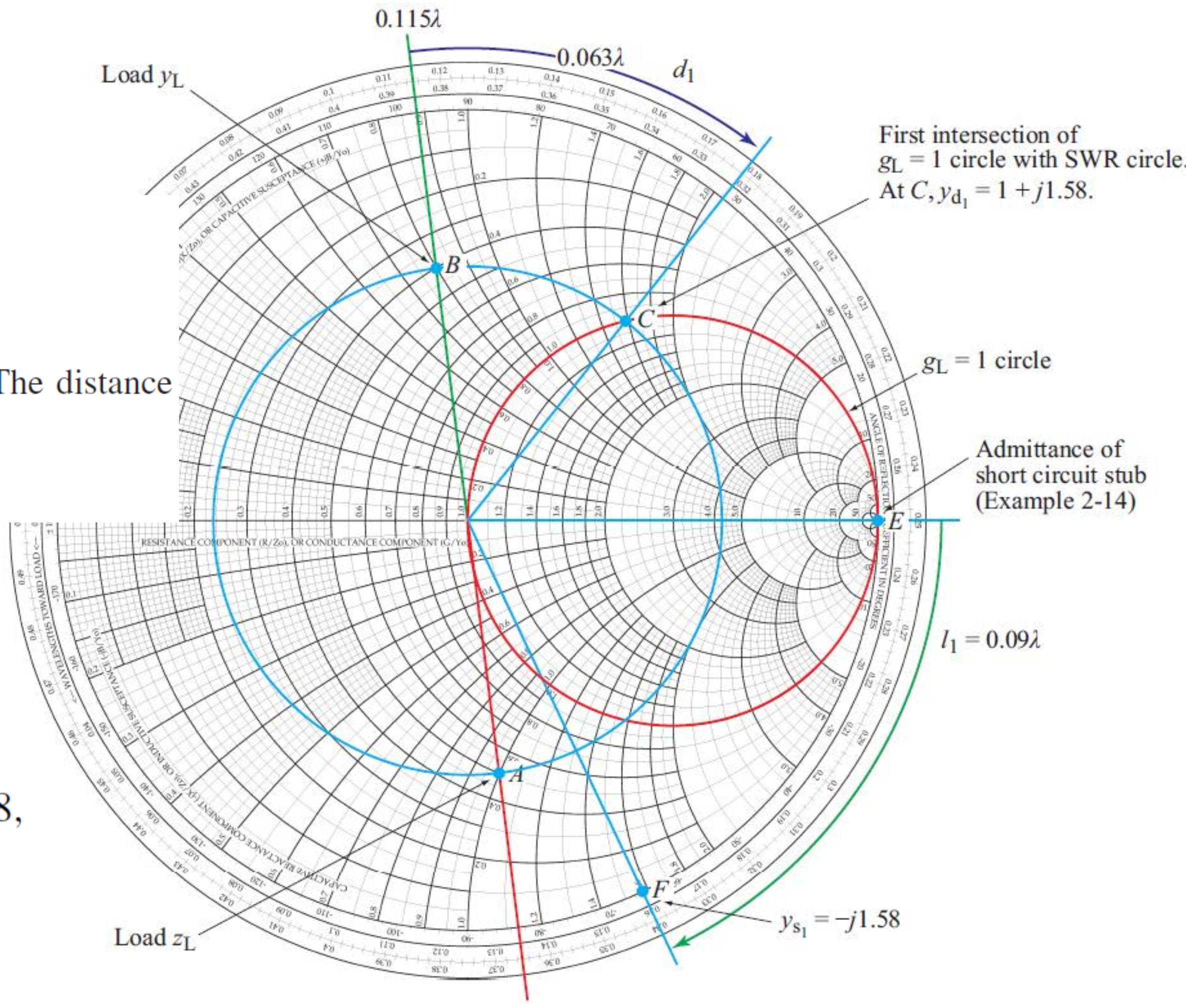
$$y_L = 0.4 + j0.8$$

Solution for Point C (Fig. 2-36): At C,

$$y_d = 1 + j1.58,$$

which is located at 0.178λ on the WTG scale. The distance between points B and C is

$$\boxed{d_1 = (0.178 - 0.115)\lambda = 0.063\lambda.}$$



we need $y_{in} = 1 + j0$. Thus,

$$1 + j0 = y_s + 1 + j1.58,$$

or

$$\boxed{y_s = -j1.58.}$$

Example 2-13: Lumped Element Cont.

A load impedance $Z_L = 25 - j50\ \Omega$ is connected to a 50-Ω transmission line. Insert a shunt element to eliminate reflections towards the sending end of the line. Specify the insert location d (in wavelengths), the type of element, and its value, given that $f = 100$ MHz.

$$z_L = \frac{Z_L}{Z_0} = \frac{25 - j50}{50} = 0.5 - j1$$

$$y_L = 0.4 + j0.8$$

Solution for Point C (Fig. 2-36): At C,

$$y_d = 1 + j1.58,$$

which is located at 0.178λ on the WTG scale. The distance between points B and C is

$$d_1 = (0.178 - 0.115)\lambda = 0.063\lambda.$$

we need $y_{in} = 1 + j0$. Thus,

$$1 + j0 = y_s + 1 + j1.58,$$

or

$$y_s = -j1.58.$$

The corresponding impedance of the lumped element is

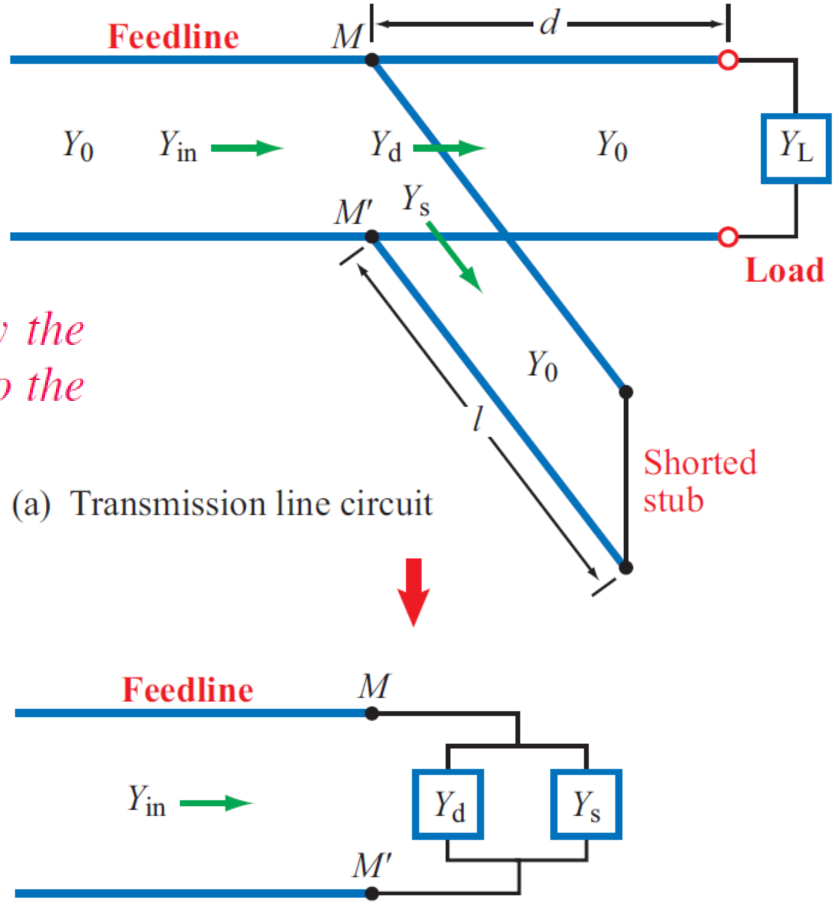
$$Z_{s_1} = \frac{1}{Y_{s_1}} = \frac{1}{y_{s_1} Y_0} = \frac{Z_0}{jb_{s_1}} = \frac{Z_0}{-j1.58} = \frac{jZ_0}{1.58} = j31.62\ \Omega.$$

Since the value of Z_{s_1} is positive, the element to be inserted should be an inductor and its value should be

$$\boxed{L = \frac{31.62}{\omega} = \frac{31.62}{2\pi \times 10^8} = 50 \text{ nH.}}$$

Single-Stub Matching

The required two degrees of freedom are provided by the length l of the stub and the distance d from the load to the stub position.



(a) Transmission line circuit

(b) Equivalent circuit

Example 2-14: Single-Stub Matching

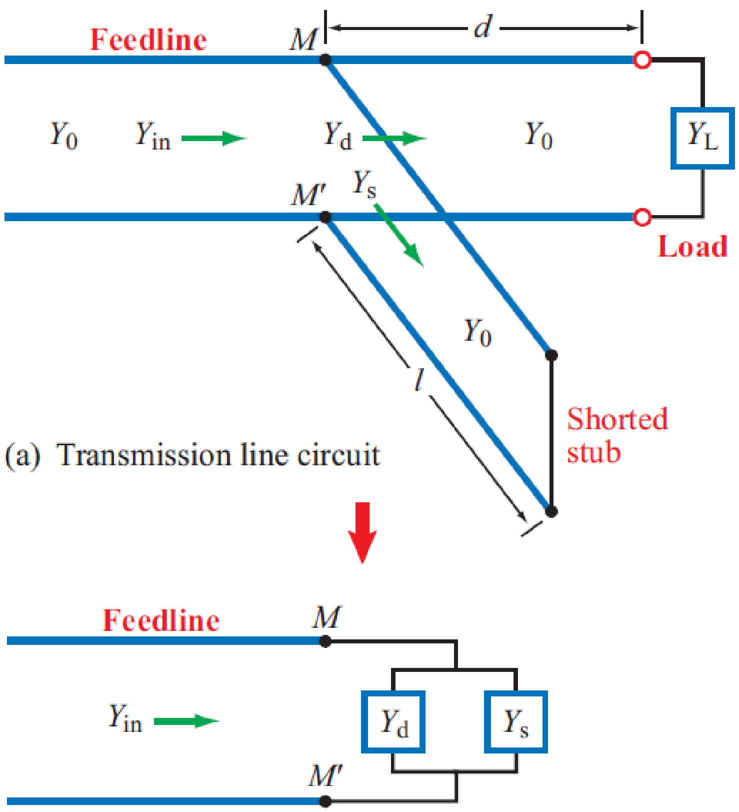
Repeat Example 2-13, but use a shorted stub (instead of a lumped element) to match the load impedance $Z_L = (25 - j50)\ \Omega$ to the 50-Ω transmission line.

Solution: In Example 2-13, we demonstrated that the load can be matched to the line via either of two solutions:

(1) $\boxed{d_1 = 0.063\lambda,}$ and $y_{s_1} = jb_{s_1} = -j1.58$,

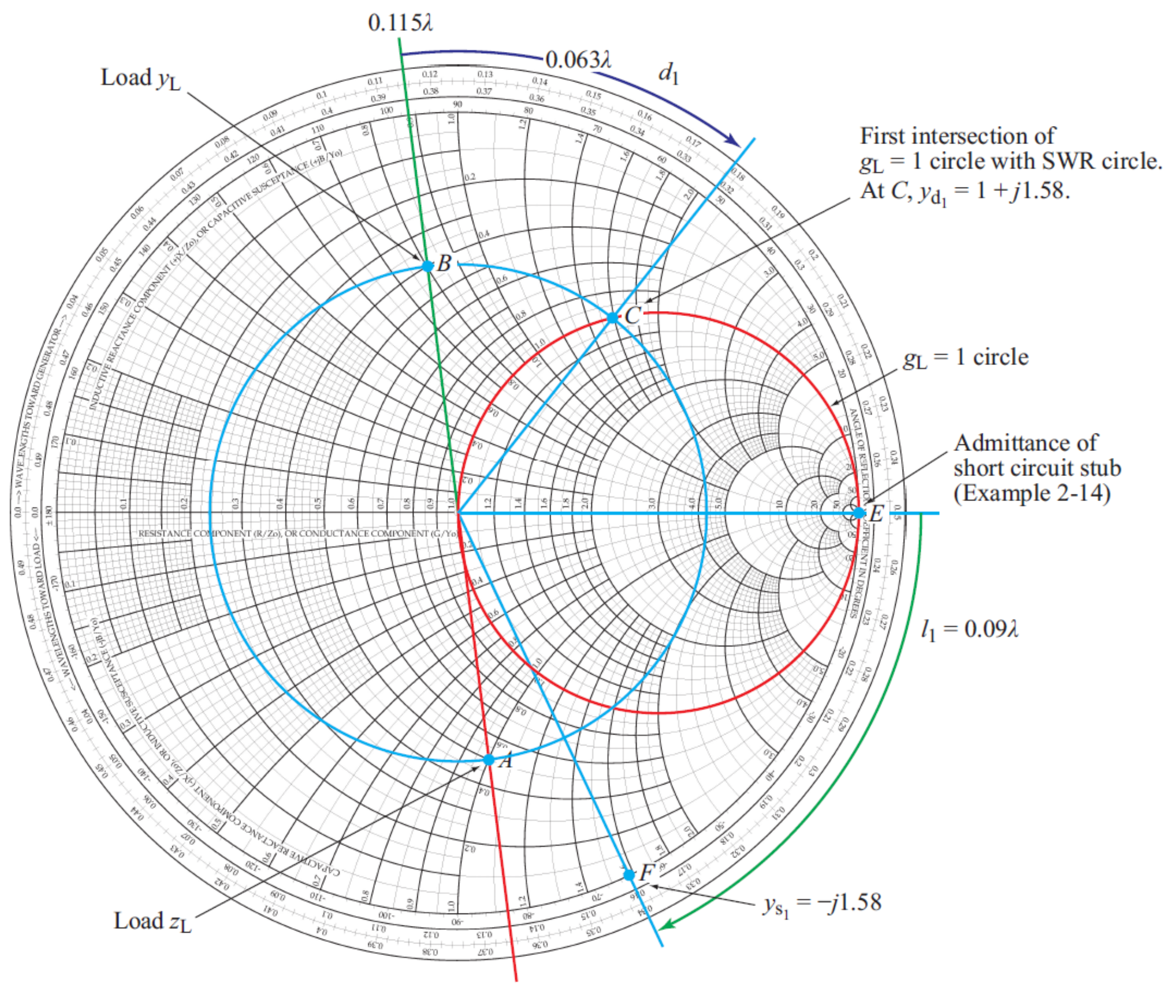
(2) $d_2 = 0.207\lambda,$ and $y_{s_2} = jb_{s_2} = j1.58$.

$$\boxed{l_1 = (0.34 - 0.25)\lambda = 0.09\lambda}$$

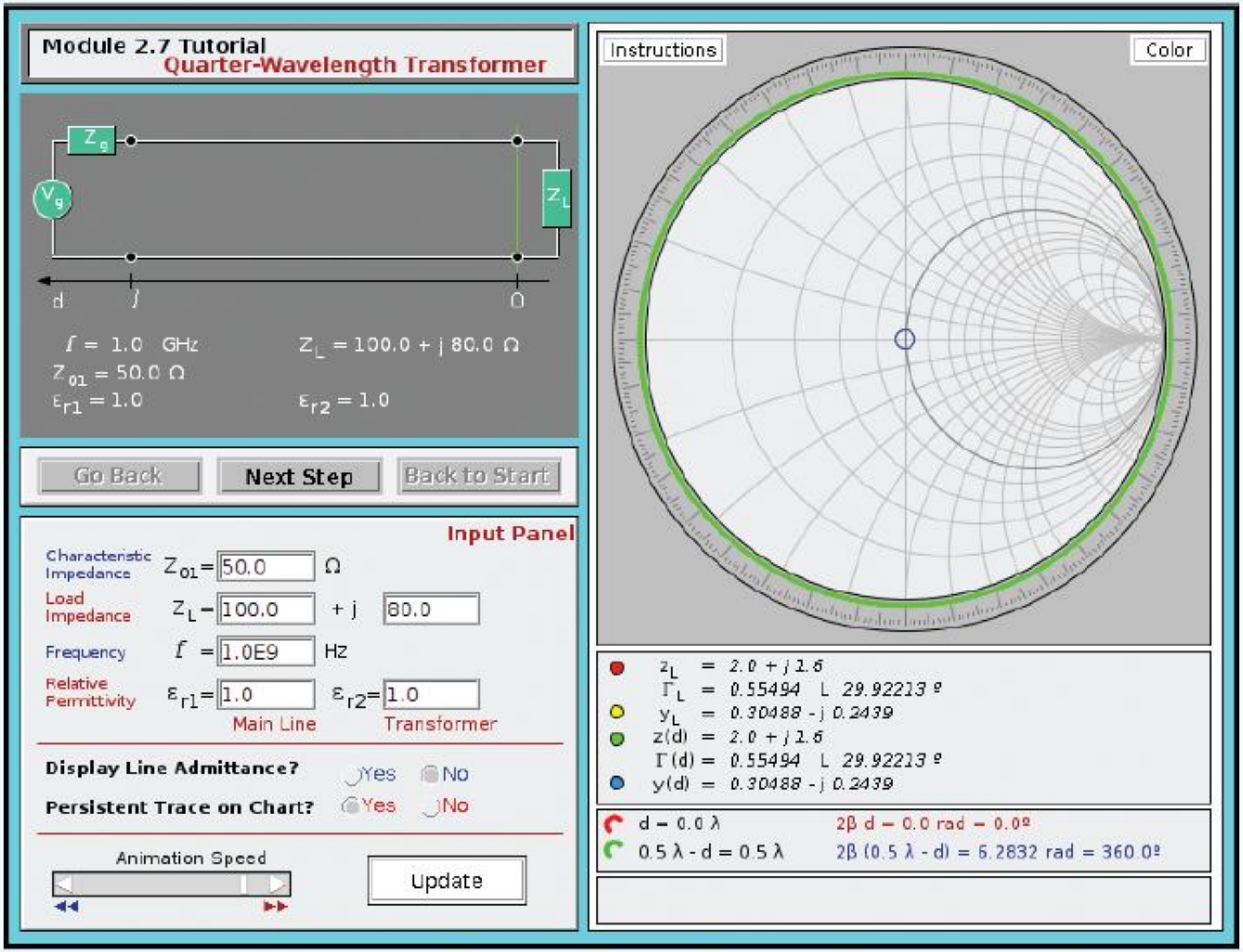


(a) Transmission line circuit

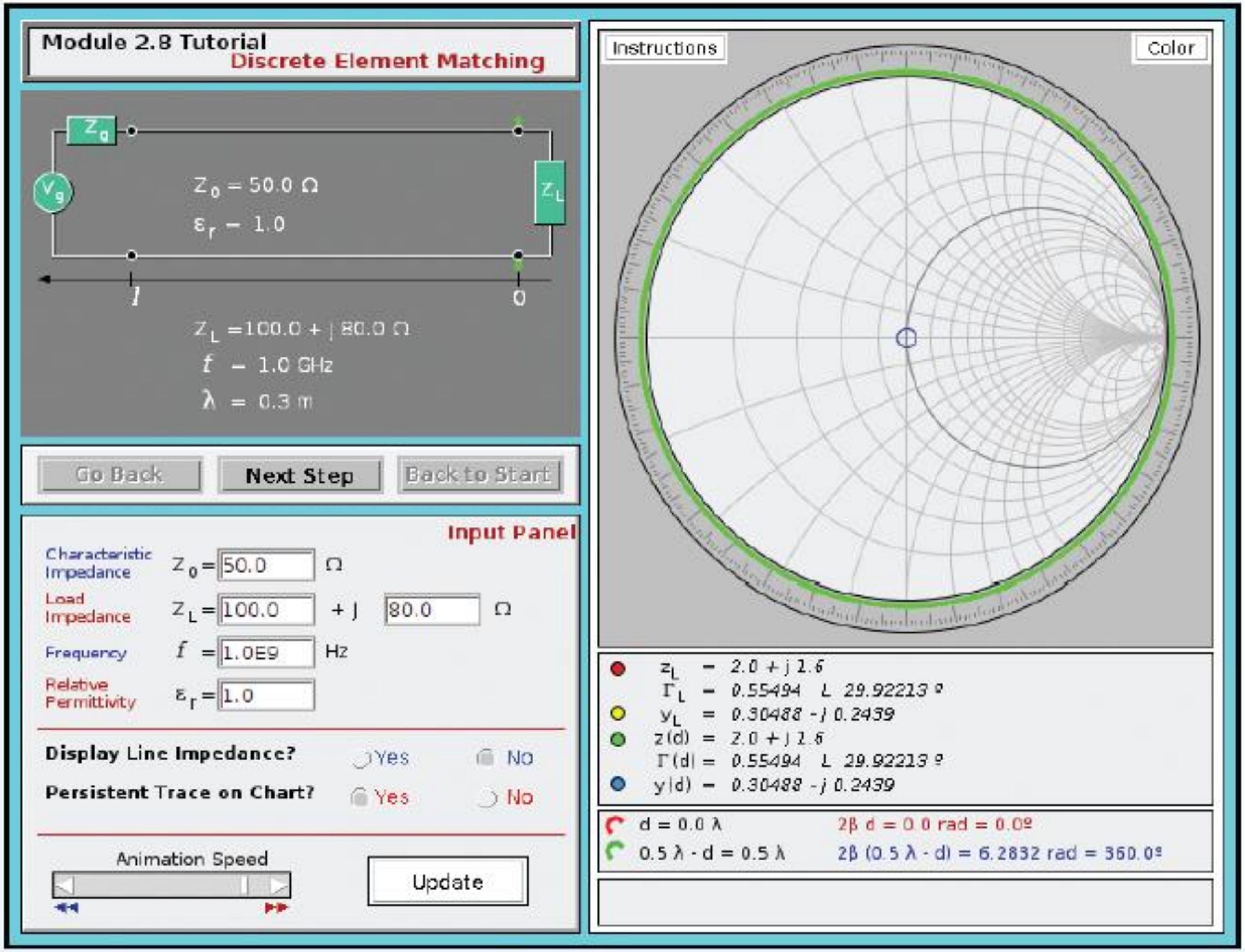
(b) Equivalent circuit



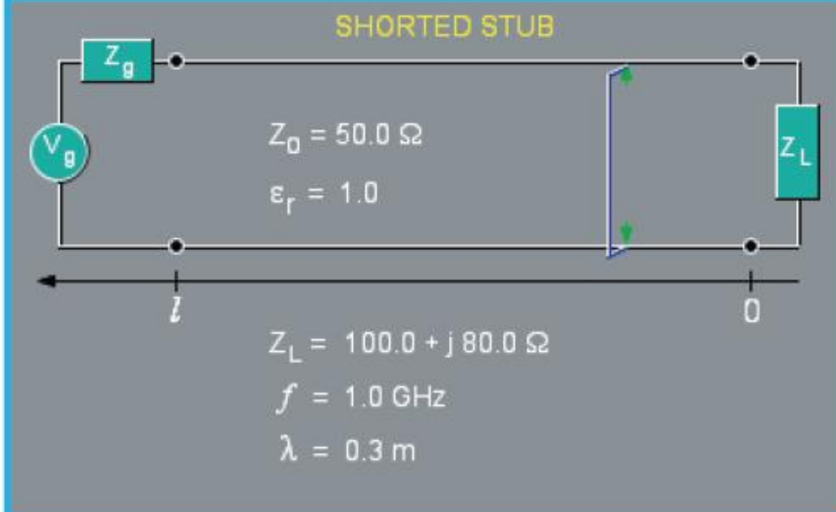
Module 2.7 Quarter-Wavelength Transformer This module allows you to go through a multi-step procedure to design a quarter-wavelength transmission line that, when inserted at the appropriate location on the original line, presents a matched load to the feedline.



Module 2.8 Discrete Element Matching For each of two possible solutions, the module guides the user through a procedure to match the feedline to the load by inserting a capacitor or an inductor at an appropriate location along the line.



Module 2.9 **Single-Stub Tuning**



Go Back | Next Step | Back to Start

Step 4 - First Solution

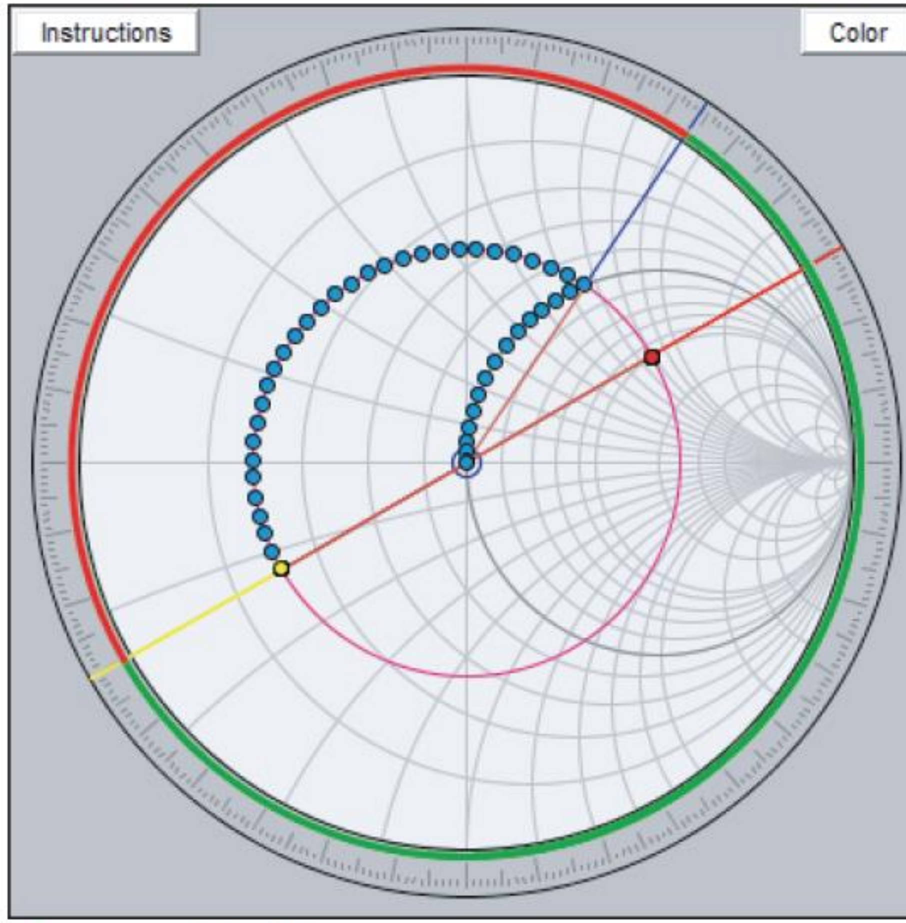
To cancel the imaginary part of the line admittance we add a stub with:

Length: $l_1 = 0.1024\ \lambda$

Admittance:

$Y_1 = -j\ 0.02668$ S (actual)

$y_1 = -j\ 1.33417$ (normalized)



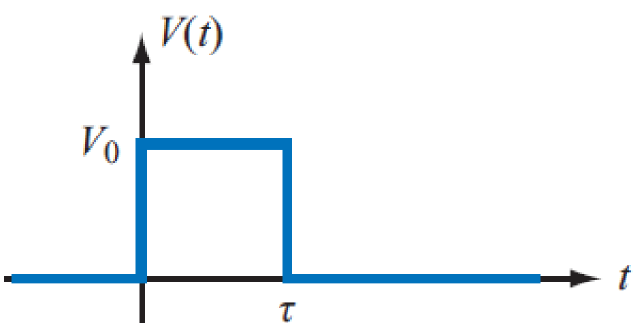
● (red)	z_L	=	$2.0 + j\ 1.6$
	Γ_L	=	0.55494 ∠ 29.92213°
● (yellow)	y_L	=	$0.30488 - j\ 0.2439$
● (green)	$z(d)$	=	$1.0 + j\ 0.0$
	$\Gamma(d)$	=	0.0 ∠ 0.0°
● (blue)	$y(d)$	=	$1.0 + j\ 0.0$

$d = 0.2134\ \lambda$ $2\beta d = 2.6813$ rad $= 153.6287°$

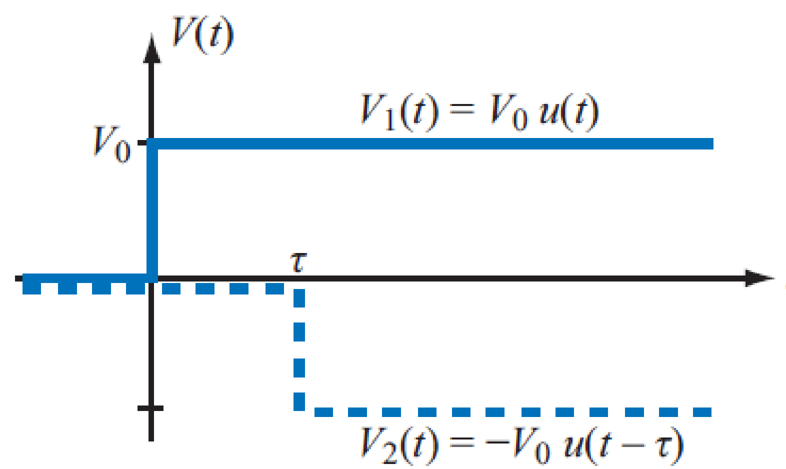
$0.5\ \lambda - d = 0.2866\ \lambda$ $2\beta\ (0.5\ \lambda - d) = 3.6019$ rad $= 206.3713°$

Transients

*The **transient response** of a voltage pulse on a transmission line is a time record of its back and forth travel between the sending and receiving ends of the line, taking into account all the multiple reflections (echoes) at both ends.*



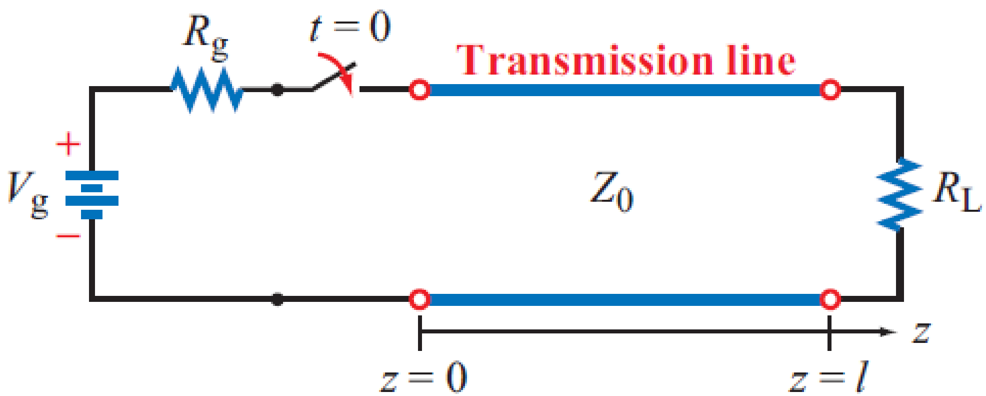
(a) Pulse of duration τ



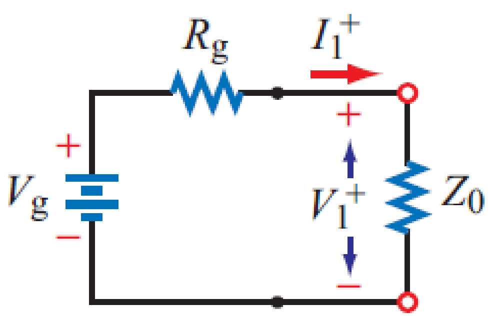
(b) $V(t) = V_1(t) + V_2(t)$

Rectangular pulse is equivalent to the sum of two step functions

Transient Response



(a) Transmission-line circuit



(b) Equivalent circuit at $t = 0^+$

Initial current and voltage

$$I_1^+ = \frac{V_g}{R_g + Z_0},$$

$$V_1^+ = I_1^+ Z_0 = \frac{V_g Z_0}{R_g + Z_0}$$

Reflection at the load

$$V_1^- = \Gamma_L V_1^+,$$

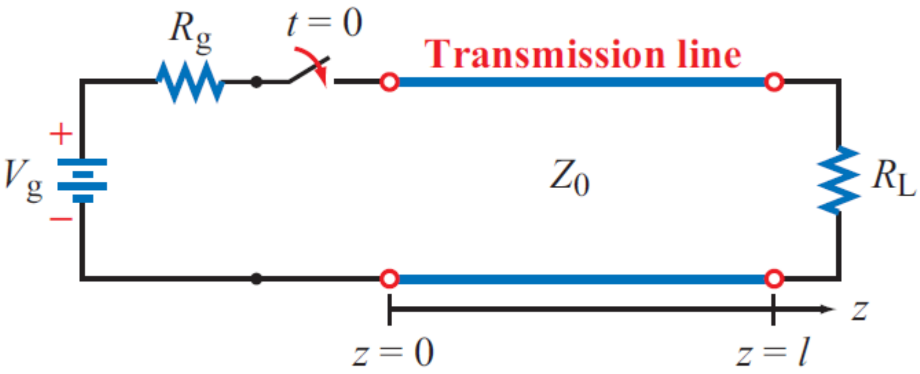
Load reflection coefficient $\Gamma_L = \dfrac{R_L - Z_0}{R_L + Z_0}$

Second transient

$$V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+$$

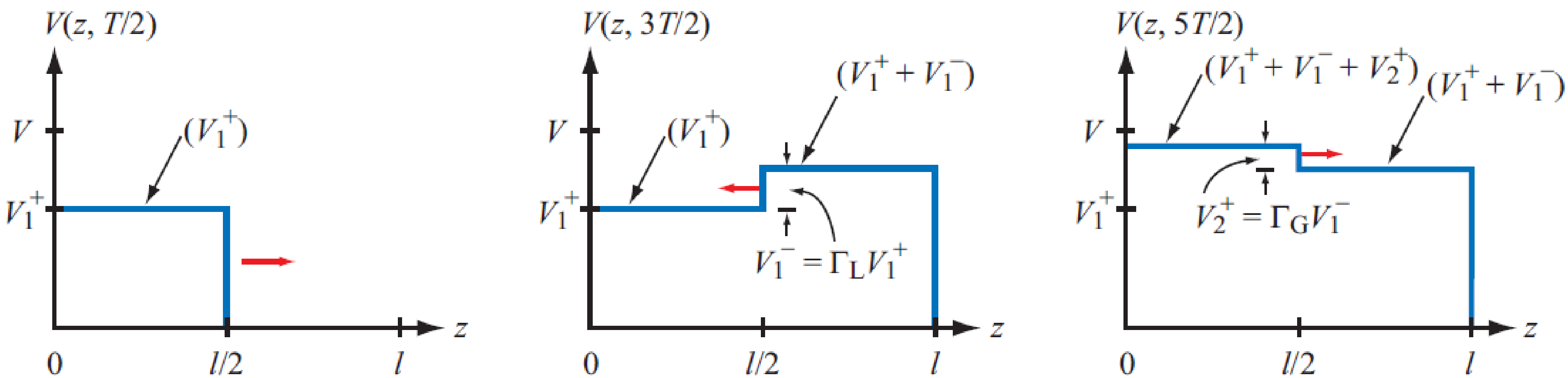
Generator reflection coefficient $\Gamma_g = \dfrac{R_g - Z_0}{R_g + Z_0}$

Voltage Wave



(a) Transmission-line circuit

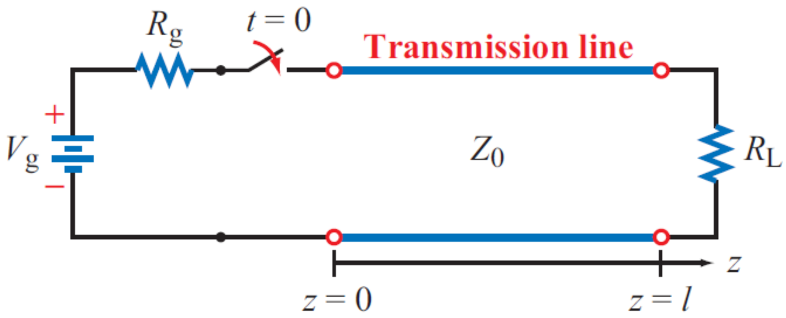
$T = l/u_p$ is the time it takes the wave to travel the full length of the line



(a) $V(z)$ at $t = T/2$ (b) $V(z)$ at $t = 3T/2$ (c) $V(z)$ at $t = 5T/2$

$R_g = 4Z_0$ and $R_L = 2Z_0$. The corresponding reflection coefficients are $\Gamma_L = 1/3$ and $\Gamma_g = 3/5$.

Current Wave

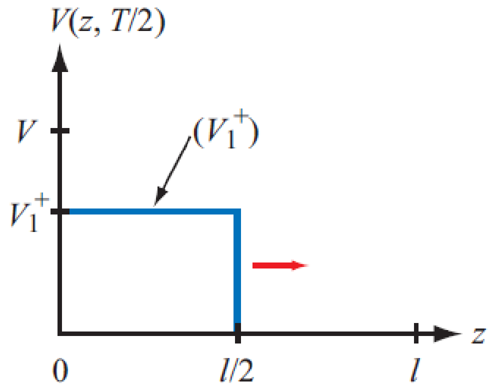


(a) Transmission-line circuit

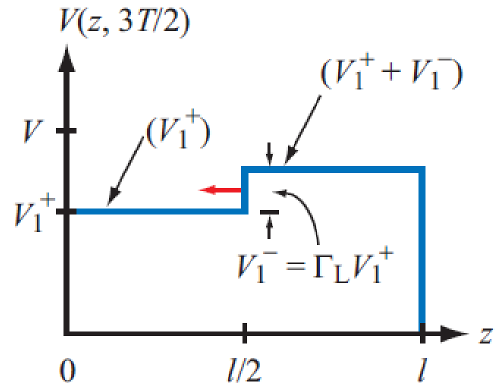
Reflection coefficient for current is the negative of that for voltage

$$I_1^- = -\Gamma_L I_1^+,$$

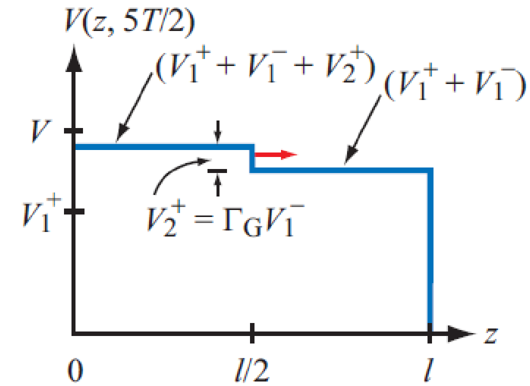
$$I_2^+ = -\Gamma_g I_1^- = \Gamma_g \Gamma_L I_1^+$$



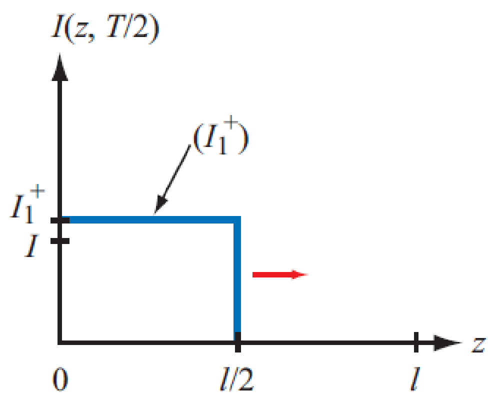
(a) $V(z)$ at $t = T/2$



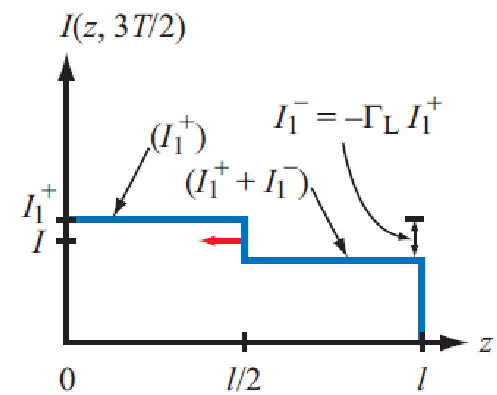
(b) $V(z)$ at $t = 3T/2$



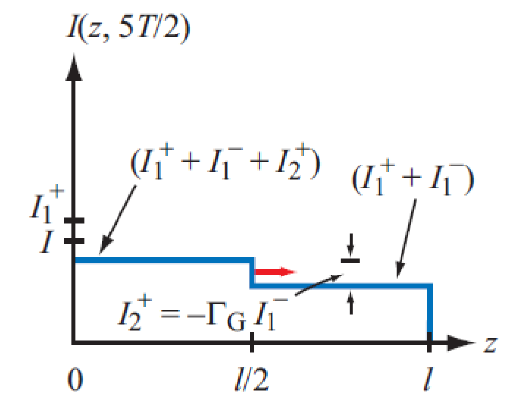
(c) $V(z)$ at $t = 5T/2$



(d) $I(z)$ at $t = T/2$

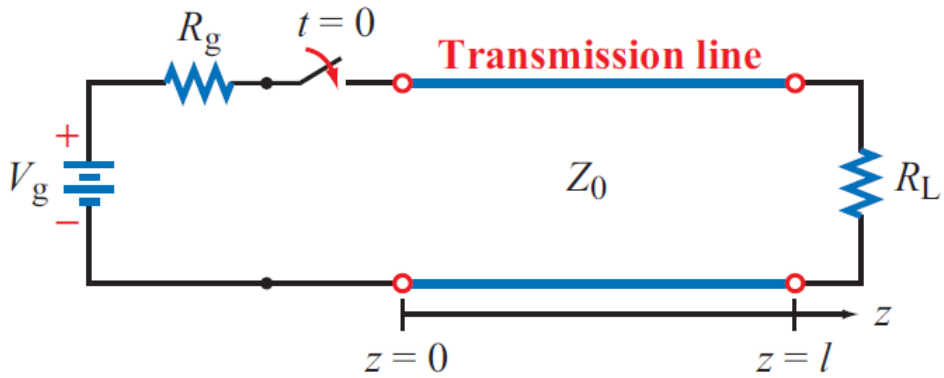


(e) $I(z)$ at $t = 3T/2$



(f) $I(z)$ at $t = 5T/2$

Steady State Response



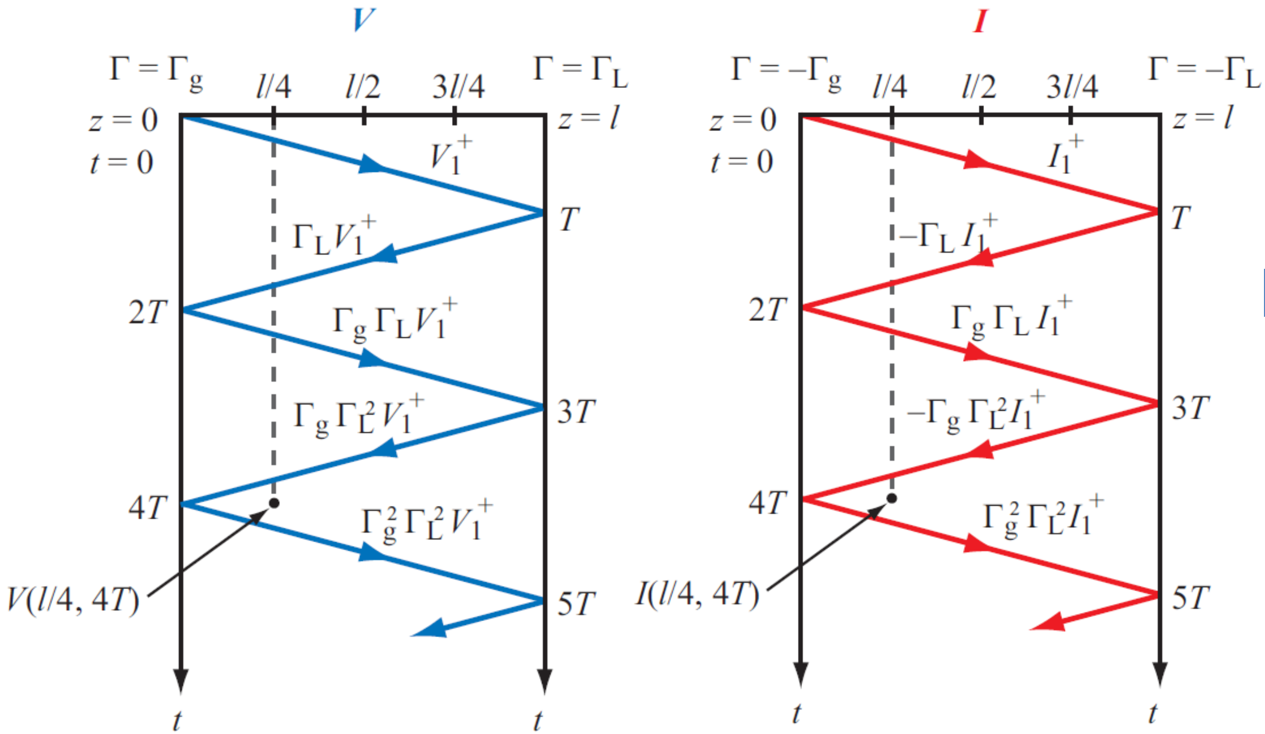
(a) Transmission-line circuit

$$V_\infty = \frac{V_g R_L}{R_g + R_L}$$

The multiple-reflection process continues indefinitely, and the ultimate value that $V(z, t)$ reaches as t approaches $+\infty$ is the same at all locations on the transmission line.

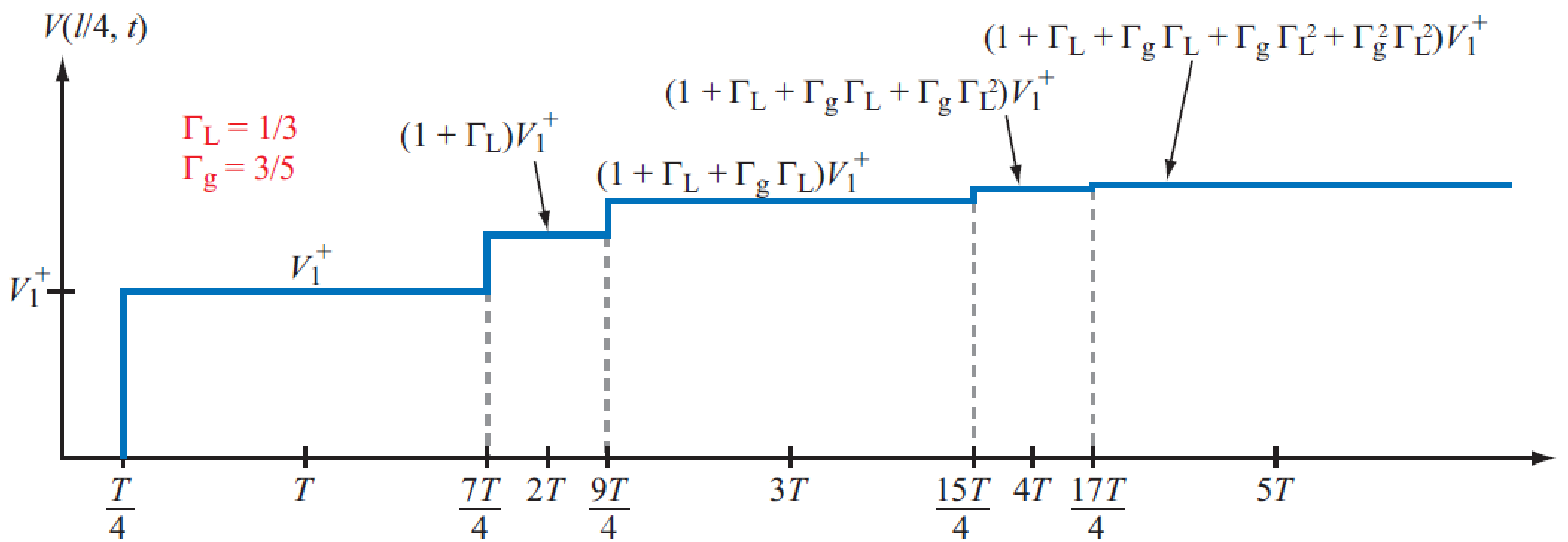
$$I_\infty = \frac{V_\infty}{R_L} = \frac{V_g}{R_g + R_L}$$

Bounce Diagrams



(a) Voltage bounce diagram

(b) Current bounce diagram



(c) Voltage versus time at $z = l/4$

Example 2-15: Pulse Propagation

The transmission-line circuit of Fig. 2-43(a) is excited by a rectangular pulse of duration $\tau = 1$ ns that starts at $t = 0$. Establish the waveform of the voltage response at the load, given that the pulse amplitude is 5 V, the phase velocity is c, and the length of the line is 0.6 m.

Solution: The one-way propagation time is

$$T = \frac{l}{c} = \frac{0.6}{3 \times 10^8} = 2 \text{ ns}.$$

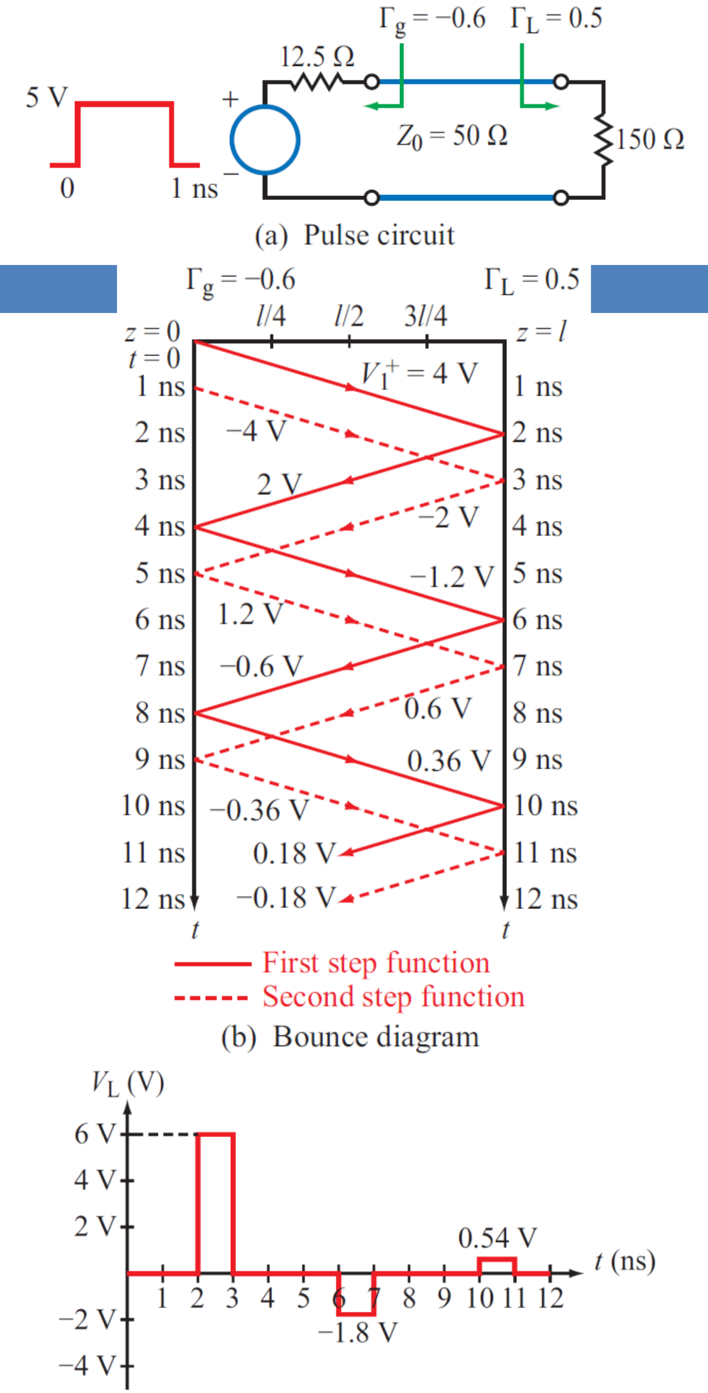
The reflection coefficients at the load and the sending end are

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = \frac{150 - 50}{150 + 50} = 0.5,$$

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{12.5 - 50}{12.5 + 50} = -0.6.$$

By Eq. (2.147), the pulse is treated as the sum of two step functions, one that starts at $t = 0$ with an amplitude $V_{10} = 5$ V and a second one that starts at $t = 1$ ns with an amplitude $V_{20} = -5$ V. Except for the time delay of 1 ns and the sign reversal of all voltage values, the two step functions will generate identical bounce diagrams, as shown in Fig. 2-43(b). For the first step function, the initial voltage is given by

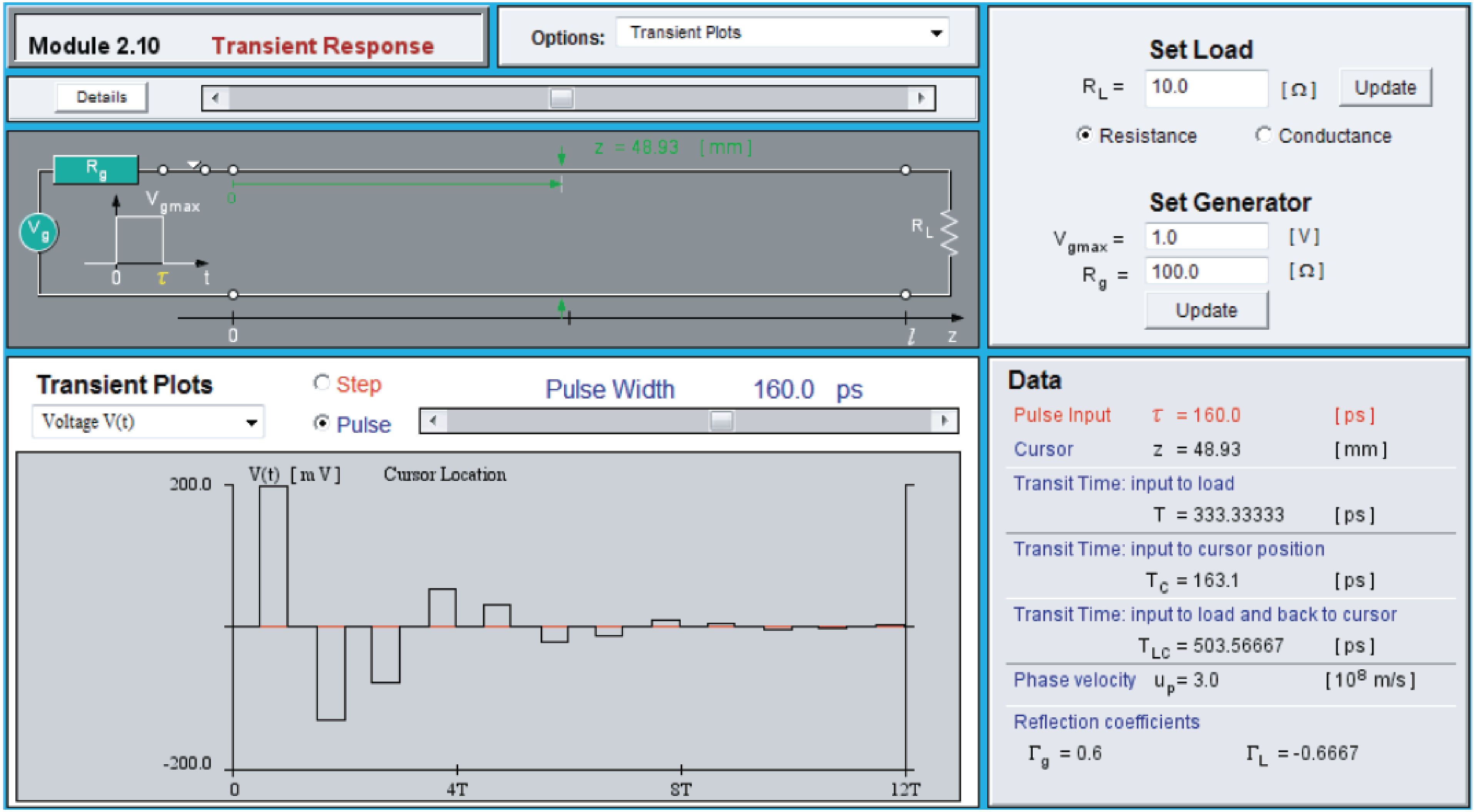
$$V_1^+ = \frac{V_{01} Z_0}{R_g + Z_0} = \frac{5 \times 50}{12.5 + 50} = 4 \text{ V}.$$



(a) Pulse circuit

(b) Bounce diagram

(c) Voltage waveform at the load



Module 2.10 Transient Response
Options: Transient Plots
Details
z = 48.93 [mm]
R_g
V_g
V_{gmax}
0
τ
t
R_L
0
l
z
Set Load
R_L = 10.0 [Ω] Update
Resistance
Conductance
Set Generator
V_{gmax} = 1.0 [V]
R_g = 100.0 [Ω]
Update
Transient Plots
Voltage V(t)
Step
Pulse
Pulse Width 160.0 ps
200.0
V(t) [mV]
Cursor Location
-200.0
0
4T
8T
12T
Data
Pulse Input τ = 160.0 [ps]
Cursor z = 48.93 [mm]
Transit Time: input to load
T = 333.33333 [ps]
Transit Time: input to cursor position
T_c = 163.1 [ps]
Transit Time: input to load and back to cursor
T_{LC} = 503.56667 [ps]
Phase velocity u_p = 3.0 [10^8 m/s]
Reflection coefficients
Γ_g = 0.6
Γ_L = -0.6667

Technology Brief 4: EM Cancer Zapper

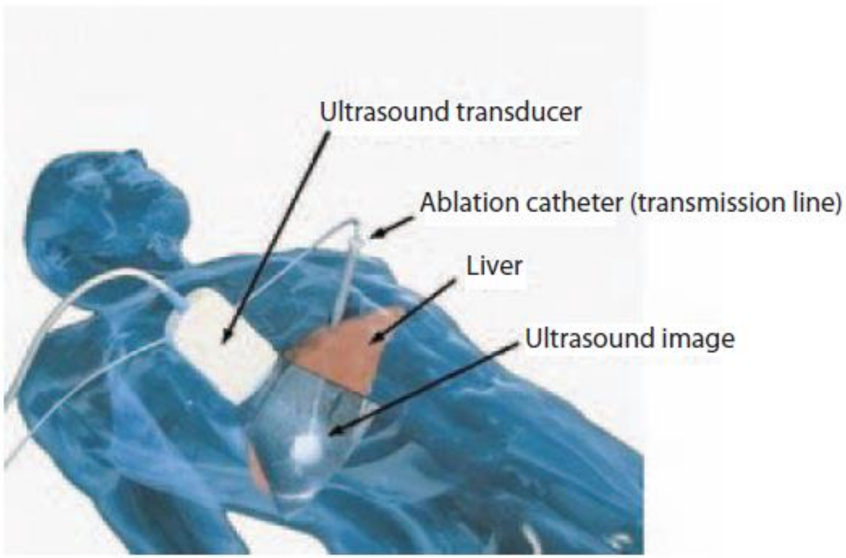
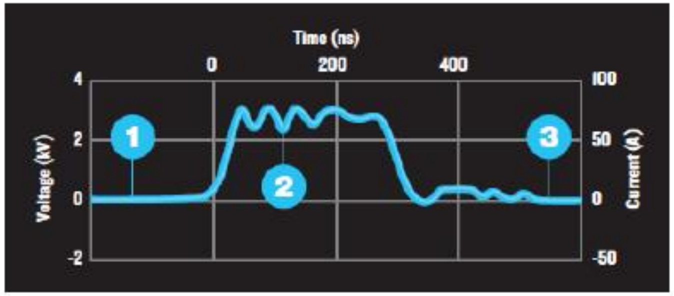


Figure TF4-1: Microwave ablation for liver cancer treatment.

Figure TF4-2: Photograph of the setup for a percutaneous microwave ablation procedure in which three single microwave applicators are connected to three microwave generators. (Courtesy of *RadioGraphics*, October 2005 pp. 569–583.)

Technology Brief 4: High Voltage Pulses



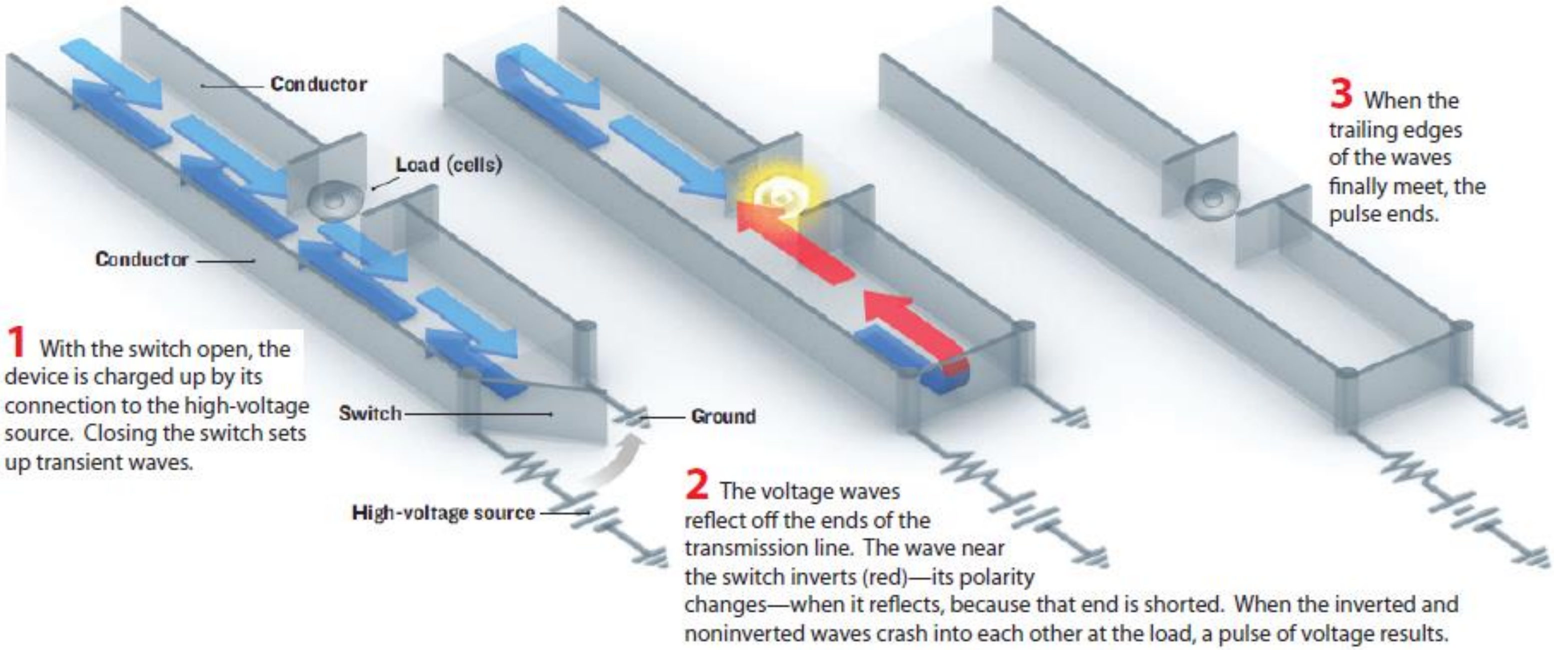


Figure TF4-3: High-voltage nanosecond pulse delivered to tumor cells via a transmission line. The cells to be shocked by the pulse sit in a break in one of the transmission-line conductors. (Courtesy of *IEEE Spectrum*, August 2006.)

Summary

Chapter 2 Relationships

TEM Transmission Lines

$$L'C' = \mu\varepsilon$$

$$\frac{G'}{C'} = \frac{\sigma}{\varepsilon}$$

$$\alpha = \mathfrak{Re}(\gamma) = \mathfrak{Re}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad \text{(Np/m)}$$

$$\beta = \mathfrak{Im}(\gamma) = \mathfrak{Im}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad \text{(rad/m)}$$

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \quad (\Omega)$$

$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

Step Function Transient Response

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0}$$

$$V_\infty = \frac{V_g R_L}{R_g + R_L}$$

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$

Lossless Line

$$\alpha = 0$$

$$\beta = \omega\sqrt{L'C'}$$

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

$$u_p = \frac{1}{\sqrt{\mu\varepsilon}} \quad \text{(m/s)}$$

$$\lambda = \frac{u_p}{f} = \frac{c}{f}\frac{1}{\sqrt{\varepsilon_r}} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

$$d_{max} = \frac{\theta_r\lambda}{4\pi} + \frac{n\lambda}{2}$$

$$d_{min} = \frac{\theta_r\lambda}{4\pi} + \frac{(2n+1)\lambda}{4}$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$P_{av} = \frac{|V_0^+|^2}{2Z_0}\,[1 - |\Gamma|^2]$$