The Cascode Amplifier

- A two transistor amplifier used to obtain simultaneously:
  1. Reasonably high input impedance.
  2. Reasonable voltage gain.
  3. Wide bandwidth.

- None of the conventional single transistor designs will satisfy all of the criteria above.
- The cascode amplifier will satisfy all of these criteria.
- A cascode is a CE Stage cascaded with a CB Stage.

(Historical Note: the cascode amplifier was a cascade of grounded cathode and grounded grid vacuum tube stages – hence the name “cascode,” which has remained in modern terminology.)
The Cascode Amplifier

Comments:
1. $R_1$, $R_2$, $R_3$, and $R_C$ set the bias levels for both $Q_1$ and $Q_2$.
2. Determine $R_E$ for the desired voltage gain.
3. $C_{in}$ and $C_{byp}$ are to act as “open circuits” at dc and act as “short circuits” at all operating frequencies $f > f_{min}$. 
Cascode Mid-Band Small Signal Model

CB Stage

CE Stage

VByp

RC

RB = R2 || R3

VGCC

iC1

iB1

iB2

iE1

iE2

vO

vbe1

vbe2

gm1

gm2

rπ1

rπ2

RS

RB

RE

B1

B2

E1

E2

Q1

Q2

C1

C2

Cin

Vo

Rs

Vs

VCC

Vbyp

CB Stage

CE Stage
Cascode Small Signal Analysis

1. Show reduction in Miller effect
2. Evaluate small-signal voltage gain

OBSERVATIONS

a. The emitter current of the CB Stage is the collector current of the CE Stage. (This also holds for the dc bias current.)

\[ i_{e1} = i_{c2} \]

b. The base current of the CB Stage is:

\[ i_{b1} = \frac{i_{e1}}{\beta + 1} = \frac{i_{c2}}{\beta + 1} \]

c. Hence, both stages have about same collector current \( i_{c1} \approx i_{c2} \) and same \( g_m, r_e, r_\pi \).
The input resistance $R_{in1}$ to the CB Stage is the small-signal "$r_{e1}$" for the CB Stage, i.e.

$$i_{bl} = \frac{i_{e1}}{\beta + 1} = \frac{i_{c2}}{\beta + 1}$$

The CE output voltage, the voltage drop from Q2 collector to ground, is:

$$v_{c2} = v_{el} = -r_\pi i_{bl} = -\frac{r_\pi}{\beta + 1} i_{c2} = -\frac{r_\pi}{\beta + 1} i_{el}$$

Therefore, the CB Stage input resistance is:

$$R_{in1} = \frac{v_{el}}{-i_{el}} = \frac{r_\pi}{\beta + 1} = r_{e1}$$

$$A_{vCE-Stage} = \frac{v_{c2}}{v_s} \approx -\frac{R_{in1}}{R_E} = -\frac{r_e}{R_E} < 1 \Rightarrow C_{eq} = (1 + \frac{r_e}{R_E})C_\mu < 2C_\mu$$
Cascode Small Signal Analysis - cont.

Now, find the CE collector current in terms of the input voltage $v_s$:

Recall $i_{c1} \approx i_{c2}$

\[
i_{b2} \approx \frac{v_s}{R_S || R_B + r_\pi + (\beta + 1) R_E}
\]

\[
i_{c2} = \beta i_{b2} \approx \frac{\beta v_s}{R_S || R_B + r_\pi + (\beta + 1) R_E} \approx \frac{\beta v_s}{(\beta + 1) R_E}
\]

for bias insensitivity: $(\beta + 1) R_E \gg R_S || R_B + r_\pi$

\[
i_{c1} \approx i_{e1} = i_{c2} \approx i_{e2}
\]

\[
i_{c2} \approx \frac{v_s}{R_E}
\]

\[
v_o = -i_{c2} R_C
\]

\[
A_v = \frac{v_o}{v_s} = \frac{-R_C}{R_E}
\]

OBSERVATIONS:

1. Voltage gain $A_v$ is about the same as a stand-alone CE Amplifier.
2. HF cutoff is much higher than a CE Amplifier due to the reduced $C_{eq}$. 
1. Choose $I_{E1}$ – make it relatively large to reduce $R_{in1} = r_e = V_T / I_{E1}$ to push out HF break frequencies.

2. Choose $R_C$ for suitable voltage swing $V_{Cl}$ and $R_E$ for desired gain.

3. Choose bias resistor string such that its current $I_I$ is about 0.1 of the collector current $I_{C1}$.

4. Given $R_E$, $I_{E2}$ and $V_{BE2} = 0.7$ V calc. $R_3$.

5. Need to also determine $R_1$ & $R_2$. 

\[ \alpha_2 I_{E2} = I_{C2} = I_{E1} = \frac{1}{\alpha_1} I_{C1} \Rightarrow I_{C1} \approx I_{E2} \]
Cascode Biasing - cont.

Since the CE-Stage gain is very small:

a. The collector swing of Q2 will be small.
b. The Q2 collector bias \( V_{C2} = V_{B1} - 0.7 \, V \).

6. Set \( V_{B1} - V_{B2} = 1 \, V \Rightarrow V_{CE2} = 1 \, V \)

This will limit \( V_{CB2} = V_{CE2} - V_{BE2} = 0.3 \, V \)
which will keep Q2 forward active.

7. Next determine \( R_2 \). Its drop \( V_{R2} = 1 \, V \)
with the known current.

\[
V_{CE2} = V_{C2} - V_R = V_{C2} - (V_{B2} - 0.7 \, V) \\
= V_{B1} - 0.7 \, V - V_{B2} + 0.7 \, V \\
= V_{B1} - V_{B2}
\]
Cascode Biasing - cont.

\[ R_2 = \frac{V_{B1} - V_{B2}}{I_1} = \frac{1 V}{I_1} \]

8. Then calculate \( R_3 \).

\[ R_3 = \frac{V_{B2}}{I_1} \]

where \( V_{B2} = 0.7 V + I_E R_E \)

Note: \( R_1 + R_2 + R_3 = \frac{V_{CC}}{I_1} \)

9. Then calculate \( R_1 \).

\[ R_1 = \frac{V_{CC}}{0.1 I_C} - R_2 - R_3 \]
Cascode Bias Summary

SPECIFIED: $A_v$, $V_{cc}$, $V_{c1}$ (CB collector voltage);
SPECIFIED: $I_E$ (or $I_C$) directly or indirectly through $BW$.
DETERMINE: $R_C$, $R_E$, $R_1$, $R_2$ and $R_3$.
SET: $V_{B1} - V_{B2} = 1$ V $\Rightarrow V_{CE2} = 1$ V

STEP1: $R_C = \frac{V_{c1}}{I_C}$  \hspace{1cm} $R_E = \frac{R_C}{A_v}$

STEP2: $R_2 = \frac{V_{B1} - V_{B2}}{I_1} = \frac{1}{0.1 I_C}$

STEP3: $R_3 = \frac{0.7 V + I_E R_E}{I_1} = \frac{V_{CC}}{0.1 I_C}$

STEP4: $R_1 = \frac{V_{CC}}{0.1 I_C} - R_2 - R_3$
Cascode Bias Example

Typical Bias Conditions

- $V_{CE1} = V_{CC} - I_C R_C - V_{CE2} - I_C R_E$
- $V_{CC} = 12 \text{ V}$
- $I_C R_E + 0.7$
- $1.0$
- $V_{CC} - I_C R_E - 1.7$

$I_{E2} \approx I_{C2} = I_{E1} \approx I_{C1} \Rightarrow I_{C1} \approx I_{E2}$

Cascode Amp
Cascode Bias Example cont.

1. Choose $I_{E1}$ – to set $r_e$.
   Try $I_{E1} = 5 \text{ mA} \Rightarrow r_e = 0.025 \text{ V} / I_E = 5 \Omega$.

2. Set desired gain magnitude. For example if $A_V = -10$, then $R_C / R_E = 10$.

3. Since the CE stage gain is very small, $V_{CE2}$ can be small, i.e. $V_{CE2} = V_{B1} - V_{B2} = 1 \text{ V}$. 

\[ V_{CE1} = V_{CC} - I_C R_C - I_C R_E \]

\[ V_{CE2} = 1 \]

\[ Vcc = 12 \text{ V} \]
Cascode Bias Example cont.

Specs:
\[ V_{CC} = 12 \text{ V} \quad V_{C1} = 7 \text{ V} \quad I_C = 5 \text{ mA} \quad |A_v| = \frac{R_C}{R_E} = 10 \]
\[ \beta = 100 \]

Determine \( R_C \) for \( V_{C1} = 7 \text{ V} \).

\[
R_C = \frac{V_{CC} - V_{C1}}{I_C} = \frac{5 \text{ V}}{5 \times 10^{-3} \text{ A}} = 1000 \Omega
\]

\[
R_E = \frac{R_C}{|A_v|} = \frac{1000 \Omega}{10} = 100 \Omega
\]
Cascode Bias Example cont.

\[ V_{CC} = 12 \quad R_C = 1 \, k\Omega \quad I_C = 5 \, mA \quad R_E = 100 \, \Omega \]

Make current through the string of bias resistors \( I_1 = 0.1 \quad I_C = 0.5 \, mA \).

\[ \frac{V_{CC}}{I_1} = \frac{12}{5 \cdot 10^{-4}} = 24 \, k\Omega \]

Calculate the bias voltages (base side of Q1, Q2):

\[ V_{R1} = V_{CC} - I_C R_E - 1.7V = 12V - 0.5V - 1.7V = 9.8V \]

\[ V_{R2} = V_{B1} - V_{B2} = 1V \]

\[ V_{R3} = V_{B2} = I_C R_E + 0.7 = 5 \cdot 10^{-3} \cdot 100 + 0.7 = 1.2V \]
Cascode Bias Example cont.

\[ V_{B2} = 5 \cdot 10^{-4} \quad R_3 = 1.2 \text{ V} \]

\[ R_3 = 2.4 \text{ k} \Omega \]

\[ V_{B1} - V_{B2} = 5 \cdot 10^{-4} \quad R_2 = 1.0 \text{ V} \]

\[ R_2 = 2 \text{ k} \Omega \]

Recall: \( R_1 + R_2 + R_3 = 24 \text{ k} \Omega \)

\[ R_1 = 24000 - 2.400 - 2000 = 19.6 \text{ k} \Omega \]

\[ V_{CC} = 12 \text{ V} , \quad R_C = 1 \text{ k} \Omega , \quad V_{B2} = 1.2 \text{ V} , \]

\[ I_C = 5 \text{ mA} , \quad R_E = 100 \text{ } \Omega , \quad V_{B1} - V_{B2} = 1.0 \text{ V} \]
\( \beta = 100 \)

\( r_e = 5 \Omega \Rightarrow I_C = 5 \text{mA} \)

\( V_{C1} = 7 \text{V} \)

\[ |A_v| = \frac{R_C}{R_E} = 10 \]

\[ \frac{1}{2 \pi C_{\text{tot}} R_S} \]

\( C_{\text{tot}} = C_\pi + \left(1 + \frac{r_e}{R_E}\right) C_\mu \)

\( f_H = \frac{1}{2 \pi C_{\text{tot}} R_S} \)

If \( C_\pi = 12 \text{pF} \)
\( C_\mu = 2 \text{pF} \)

\( C_{\text{tot}} = 14.1 \text{pF} \)

\( f_{H_{\text{cascode}}} = 225.8 \text{MHz} \)

For CE with \( |A_v| = 10 \)

\( f_{H_{CE}} = 94 \text{MHz} \)