2.23. (a) Express the output $y(t)$ as a function of the input and the system transformations, in the form of (2.56), for the system of Figure P2.23(a).

(b) Repeat Part (a) for the system of Figure P2.23(b).

(c) Repeat Part (a) for the case that the summing junction with inputs $y_5(t)$ and $y_6(t)$ is replaced with a multiplication junction, such that its output is the product of these two signals.

(d) Repeat Part (b) for the case that the summing junction with inputs $y_5(t)$, $y_6(t)$, and $y_7(t)$ is replaced with a multiplication junction, such that its output is the product of these three signals.

2.23 (a) $y_5(t) = T_x [T_1 (x(t))]$,

$y_6(t) = T_x [T_2 (x(t))]$,

$y(t) = T_x [T_1 (x(t))] + T_x \{ T_3 [T_1 (x(t))] + T_3 [T_2 (x(t))] \}$

(b) $y(t) = T_x [T_2 (x(t))] + T_x \{ T_3 [T_1 (x(t))] + T_3 [T_2 (x(t))] \}$

(c) $y(t) = T_x [T_2 (x(t))] + T_x \{ T_3 [T_1 (x(t))] \times T_3 [T_2 (x(t))] \}$

(d) $y(t) = T_x [T_3 (x(t))] \times T_3 [T_1 (x(t))] \times T_3 [T_2 (x(t))]$
2.26. (a) Determine whether the system described by

\[ y(t) = \int_{t}^{t+1} x(\tau - \alpha) d\tau \]

(where \( \alpha \) is a constant) is

(i) memoryless. \quad (ii) invertible.

(iii) stable. \quad (iv) time invariant, and

(v) linear.

(b) For what values of the constant \( \alpha \) is the system causal?

2.26

(a) (i) has memory; (ii) not invertible; (iii) stable; (iv) time invariant; (v) linear

(b) need \( y(t) \) to only depend on \( x(t) \) values at \( t \) and \( t+1 \), therefore causal for values of \( \alpha \geq 1 \).
2.27. (a) Determine whether the system described by

\[ y(t) = \cos[x(t - 1)] \]

is

(i) memoryless,
(ii) invertible,
(iii) causal,
(iv) stable,
(v) time invariant, and
(vi) linear.

(b) Repeat Part (a) for

\[ y(t) = 3x(3t + 3). \]

(c) Repeat Part (a) for

\[ y(t) = \ln|x(t)|. \]

(d) Repeat Part (a) for

\[ y(t) = e^{t/x}. \]

(e) Repeat Part (a) for

\[ y(t) = 7x(t) + 6. \]

(f) Repeat Part (a) for

\[ y(t) = \int_{-\infty}^{t} x(\tau) d\tau. \]

(g) Repeat Part (a) for

\[ y(t) = e^{-j\omega t} \int_{-\infty}^{\infty} x(\tau) e^{-j\omega \tau} d\tau. \]

(h) Repeat Part (a) for

\[ y(t) = \int_{t-1}^{t} x(\tau) d\tau. \]
2.27(b)
   i) not memoryless (at time $t_0$ output depends on input at time $3t_0$)
   ii) invertible ($x(t) = \frac{1}{3} y(t(t-3))$
   iii) not causal ($3t_0 > t_0$ for $t_0 > 0$)
   iv) stable
   v) not time invariant ($x(t-t_0) \rightarrow 3x(3t-t_0 + 3)$ but $y(t-t_0) = 3x(3(t-t_0) + 3) = 3x(3t-3t_0 + 3)$
   vi) linear
2.27(c) system is: $y(t) = \ln(x(t))$
   i) Memoryless;
   ii) Invertible: $x(t) = e^{y(t)}$
   iii) Causal;
   iv) Not stable: for example, $y(t) = -\infty$ whenever $x(t) = 0$
   v) Time invariant;
   vi) Not linear: for example, violates additivity: $\ln(x_1(t) + x_2(t)) \neq \ln(x_1(t)) + \ln(x_2(t))$ in general.
   Scaling doesn’t work either.

2.27(e) System is: $y(t) = 7x(t) + 6$
   This system is memoryless, invertible, causal, stable, time invariant, but NOT linear: if we input $x_1(t) + x_2(t)$ we get out $7(x_1(t) + x_2(t)) + 6$, while if we input $x_1(t)$ and $x_2(t)$ separately and add them, we get $y_1(t) + y_2(t) = 7(x_1(t)) + 6 + 7(x_2(t)) + 6$, so the system violates additivity. Also violates scaling. Note that to show a system is linear you need to show it satisfies both properties (which you can do by showing that $ax_1(t) + bx_2(t) = ay_1(t) + by_2(t)$), but to show that a system is NOT linear, you only need to show that it violates at least one of these properties.
2.31. (a) Sketch the characteristic $y$ versus $x$ for the system $y(t) = |x(t)|$. Determine whether this system is

(i) memoryless, (iii) invertible,
(ii) causal, (iv) stable,
(v) time invariant, and (vi) linear.

(b) Repeat Part (a) for

$$y(t) = \begin{cases} x(t), & x \geq 0 \\ 0, & x < 0 \end{cases}$$

(b) (i) **memoryless**
(ii) $y = 0$ for $x \leq 0$, **not invertible**
(iii) **causal**
(iv) **stable**
(v) **time invariant**
(vi) $y|_{x_1=1} \neq y|_{x_1=x_2}$, **not linear**
Additional Problem:

\[ X(t) = [1] \times [u(t+1)-u(t)] + [-3t + 1] \times [u(t) - u(t-1)] + [-2] \times [u(t-1) - u(t-2)] + [t-3] \times [u(t-2) - u(t-4)] \]

\[ X(t) = u(t+1) + [-3t] \times u(t)] + [3t - 3] \times u(t-1) + [t-1] \times u(t-2) +[-t+3] \times u(t-4) \]