Quantum Mechanics – Concepts and Applications

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Preface

The fundamental idea behind any physical theory is to develop predictive power with a minimal set of experimentally tested postulates. However, historical development of a theory is not always that systematic. Different theorists and experimentalists approach the subject differently and achieve successes in different directions which gives the subject a rather "patchy" appearance. This has been particularly true for quantum mechanics. However, now that the dust has settled and physicists know quantum mechanics reasonably well, it is necessary to consolidate concepts and put together that minimal set of postulates.

The minimal set of postulates in classical mechanics is already very well known and hence it is a much easier subject to present to a student. In quantum mechanics such a set is usually not identified in text books which, I believe, is the major cause of fear of the subject among students. Very often, text books enumerate the postulates but continue to add further assumptions while solving individual problems. This is particularly disconcerting in quantum mechanics where, physical intuition being nonexistent, assumptions are difficult to justify. It is also necessary to separate the postulates from the sophisticated mathematical techniques needed to solve problems. In doing this one may draw analogies from classical mechanics where the physical postulate is Newton's second law and everything else is creative mathematics for the purpose of using this law in different circumstances. In quantum mechanics the equivalent of Newton's second law is, of course, the Schrödinger equation. However, before using the Schrödinger equation it is necessary to understand the mathematical meanings of its components e.g. the wavefunction or the state vector. This, of course, is also true for Newton's law. There one needs to understand the relatively simple concept of particle trajectories.

Some previous texts have successfully separated the mathematics from the physical principles. However, as a consequence, they have introduced so much mathematics that the physical content of the theory is lost. Such books are better used as references rather than textbooks. The present text will attempt a compromise. It will maintain the separation of the minimal set of postulates from the mathematical techniques. At the same time close contact with experiment will be maintained to avoid alienating the physics student. Mathematical rigor will also be maintained barring some exceptions where it would take the reader too far afield into mathematics.

A significantly different feature of this book is the highlighting of numerical methods. An unavoidable consequence of doing practical physics is that most realistic problems do not have analytical solutions. The traditional approach to such problems has been a process of approximation of the complex system to a simple one and then adding appropriate numbers of correction terms. This has given rise to several methods of finding correction terms and some of them will be discussed in this text. However, these techniques were originally meant for hand computation. With the advent of present day computers more direct approaches to solving complex problems are available. Hence, besides learning to solve standard analytically solvable problems, the student needs to learn general numerical techniques that would allow one to solve any problem that has a solution. This would serve two purposes. First, it makes the student confident that every well defined problem is solvable and the world does not have to be made up of close approximations of the harmonic oscillator and the hydrogen atom. Second, one very often comes up with a problem that is so far from analytically solvable problems that standard approximation methods would not be reliable. This has been my motivation in including two chapters on numerical techniques and encouraging the student to use such techniques at every opportunity. The goal of these chapters is not to provide the most accurate algorithms or to give a complete discussion of all numerical techniques known (the list would be too long even if I were to know them all). Instead, I discuss the intuitively obvious techniques and encourage students to develop their own tailor-made recipes for specific problems.

This book has been designed for a first course (two semesters) in quantum mechanics at the graduate level. The student is expected to be familiar with the physical principles behind basic ideas like the Planck hypothesis and the de Broglie hypothesis. He (or she) would also need the background of a graduate level course in classical mechanics and some working knowledge of linear algebra and differential equations.