Solutions
Chapter 3

Problem 3.5

$$S = k \ln \Omega = k \ln \left(\frac{eN}{q} \right)^q = kq(\ln N + 1 - \ln q) = \frac{kU}{\epsilon}(\ln N + 1 - \ln U + \ln \epsilon). \tag{1}$$

Hence,

$$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{k}{\epsilon}(\ln N + 1 - \ln U + \ln \epsilon) - \frac{k}{\epsilon} = \frac{k}{\epsilon}(\ln N - \ln U + \ln \epsilon), \tag{2}$$

and,

$$\ln U = -\frac{\epsilon}{kT} + \ln(\epsilon N). \tag{3}$$

This gives,

$$U = \epsilon N e^{-\epsilon/kT}. \tag{4}$$

Problem 10

Part a

Heat acquired by melting of ice is,

$$Q_m = lm = 333 \times 30 = 9.99 \times 10^3 \text{J}. \tag{5}$$

Resulting change in entropy is,

$$\Delta S_m = \frac{Q_m}{T} = \frac{9.99 \times 10^3}{273} = 36.6 \text{ J/K} \tag{6}$$

Part b

Entropy change in heating water from 0°C to 25°C is,

$$\Delta S_w = \int dS = \int_{T_i}^{T_f} C_V \frac{dT}{T} = C_V \ln \frac{T_f}{T_i} = 4.2 \times 30 \times \ln \frac{298}{273} = 11.0 \text{ J/K}. \tag{7}$$

Part c

Heat lost by kitchen in melting ice is Q_m as calculated earlier. Heat lost by kitchen in heating water to 25°C is,

$$Q_w = C_V(T_f - T_i) = 4.2 \times 30 \times (298 - 273) = 3.15 \times 10^3 \text{ J}. \tag{8}$$

As kitchen is always at 25°C, its change in entropy is,

$$\Delta S_k = \frac{-Q_m - Q_w}{T_k} = \frac{-9.99 \times 10^3 - 3.15 \times 10^3}{298} = -44.1 \text{ J/K}. \tag{9}$$

Part d

Net change in entropy is,

$$\Delta S = \Delta S_m + \Delta S_w + \Delta S_k = 36.6 + 11.0 - 44.1 = 3.5 \text{ J/K}. \tag{10}$$

Problem 3.28

$$(\Delta S)_P = \int_{T_i}^{T_f} \frac{C_P}{T} dT = C_P \int_{T_i}^{T_f} \frac{dT}{T} = C_P \ln \frac{T_f}{T_i}. \tag{11}$$

As air is made up of mostly diatomic molecules,

$$C_P = C_V + Nk = 5Nk/2 + Nk = 7Nk/2. \tag{12}$$

Using the ideal gas law and the values for P, V and T in this problem,

$$Nk = \frac{PV}{T} = \frac{1.013 \times 10^5 \times 1.00 \times 10^{-3}}{300} = 0.338 \text{ J/K}. \tag{13}$$

Hence,

$$C_P = 7Nk/2 = \frac{7 \times 0.338}{2} = 1.18 \text{ J/K}. \tag{14}$$

Once again from the ideal gas law,

$$\frac{PV_i}{T_i} = \frac{PV_f}{T_f}. \tag{15}$$

As it is given that $V_f/V_i = 2$, this gives,

$$\frac{T_f}{T_i} = 2. \tag{16}$$

Using this and the value from equation 14 in equation 11, we get,

$$(\Delta S)_P = 1.18 \times \ln \frac{T_f}{T_i} = 1.18 \times \ln 2 = 0.820 \text{ J/K}. \tag{17}$$