Solutions
Chapter 1

Problem 1.7

Part a

Consider the bulb to have a volume of $V = 0.1$ cc and a rise in temperature of 1 K to produce a change in liquid height of $h = 5$ mm. If the inside diameter of the tube is d then the increase in volume of liquid per K is,

$$\Delta V = \pi h (d/2)^2. \tag{1}$$

As

$$\beta = \frac{\Delta V/V}{\Delta T}, \quad \Delta V = \beta V \Delta T. \tag{2}$$

Hence,

$$\pi h (d/2)^2 = \beta V \Delta T, \tag{3}$$

and,

$$d = 2\sqrt{\frac{\beta V \Delta T}{\pi h}} = 6.79 \times 10^{-5} \text{ m} = 0.0679 \text{ mm}. \tag{4}$$

Here, $\beta = 1.81 \times 10^{-4}$ K^{-1}, $V = 0.1 \times 10^{-6}$ m^3, $\Delta T = 1$ K and $h = 5 \times 10^{-3}$ m.

Problem 1.8

Part a

Let the temperature on a cold winter night be 0°F and that on a hot summer day be 90°F. This 90F° difference is the same as $90 \times 5/9 = 50$C° $= 50$K. Hence, $\Delta T = 50K$. This gives,

$$\Delta L = \alpha L \Delta T = 1.1 \times 10^{-5} \times 1000 \times 50 = 0.55 \text{ m}. \tag{5}$$

Part c

The linear expansion coefficients in the x, y and z directions can be written as,

$$\alpha_x = \frac{\Delta L_x/L_x}{\Delta T}, \quad \alpha_y = \frac{\Delta L_y/L_y}{\Delta T}, \quad \alpha_z = \frac{\Delta L_z/L_z}{\Delta T}. \tag{6}$$

This gives,

$$\Delta L_x = \alpha_x L_x \Delta T, \quad \Delta L_y = \alpha_y L_y \Delta T, \quad \Delta L_z = \alpha_z L_z \Delta T. \tag{7}$$

So, the increased volume is,

$$\begin{aligned} V + \Delta V &= (L_x + \Delta L_x)(L_y + \Delta L_y)(L_z + \Delta L_z) && (8)\\ &= L_x L_y L_z (1 + \Delta L_x/L_x)(1 + \Delta L_y/L_y)(1 + \Delta L_z/L_z) && (9)\\ &= L_x L_y L_z (1 + \alpha_x \Delta T)(1 + \alpha_y \Delta T)(1 + \alpha_z \Delta T) && (10)\\ & && (11) \end{aligned}$$

The expansion coefficients being much smaller than 1, the terms involving their products are even smaller and can be ignored in the expansion of the above expression. Hence,

$$V + \Delta V = L_x L_y L_z (1 + \alpha_x \Delta T + \alpha_y \Delta T + \alpha_z \Delta T). \tag{12}$$

As $V = L_x L_y L_z$, this gives,

$$\Delta V = (\alpha_x + \alpha_y + \alpha_z) V \Delta T \tag{13}$$

As, by definition of β,

$$\Delta V = \beta V \Delta T, \tag{14}$$

we get,

$$\beta = \alpha_x + \alpha_y + \alpha_z. \tag{15}$$

Problem 1.12

From the ideal gas law, the volume per molecule is,

$$V/N = kT/P = \frac{1.38 \times 10^{-23} \times 300}{1.01 \times 10^5} = 4.10 \times 10^{-26} \text{ m}^3. \tag{16}$$

The average distance between molecules would be the cube root of this.

$$d = (V/N)^{1/3} = 3.45 \times 10^{-9} \text{ m}. \tag{17}$$

The size of nitrogen or water molecule is about a tenth of this.

Problem 1.16

Part a

If the area of the slab is A, it volume is $dV = A\, dz$. Then, if its density is ρ, its weight is,

$$dF = g\rho dV = g\rho A\, dz, \tag{18}$$

where g is the acceleration due to gravity. So, the pressure decreases with height z by the amount of pressure produced by this weight (dF/A). Hence,

$$dP = -dF/A = -g\rho dz. \tag{19}$$

And,

$$\frac{dP}{dz} = -g\rho. \tag{20}$$

Part b

Density is mass divided by volume. The mass of N molecules of mass m each is mN. Hence, the density of the gas is,

$$\rho = mN/V. \tag{21}$$

Then, finding N/V from the gas law gives,

$$\rho = \frac{mP}{kT} \tag{22}$$

Then, using the result of the last section, we get,

$$\frac{dP}{dz} = -\frac{mg}{kT}P. \tag{23}$$

Part c

The lat equation rewritten and integrated gives,

$$\int_{P(0)}^{P} \frac{dP}{P} = -\frac{mg}{kT}\int_0^z dz, \tag{24}$$

where $P(0)$ is the pressure at $z = 0$ and P is the pressure at any height z. Performing this integral gives,

$$\ln(P/P(0)) = -\frac{mg}{kT}z. \tag{25}$$

Hence,

$$P = P(0)e^{-mgz/kT} \tag{26}$$

As seen earlier (equation 22), density is proportional to pressure and hence,

$$\rho = \rho(0)e^{-mgz/kT}, \tag{27}$$

where $\rho(0)$ is the pressure at $z = 0$.

Problem 1.22

Part a

Consider the x direction to be perpendicular to the surface. So, in an elastic collision of a molecule with the wall the x component of its momentum switches direction. Hence its x component of momentum changes by the amount $2m\overline{v_x}$. If ΔN molecules hit the surface, the total change in x component of momentum would be $2\Delta N\ m\overline{v_x}$. If this happens over a time period of Δt, the force generated in the x direction is (as force is the time rate of change of momentum),

$$F = 2\Delta N\ m\overline{v_x}/\Delta t. \tag{28}$$

As pressure P is force per unit area of the surface,

$$P = F/A = \frac{2\Delta N\ m\overline{v_x}}{A\Delta t} \tag{29}$$

Solving for ΔN the number of molecules that hit the surface in time Δt gives,

$$\Delta N = \frac{PA\Delta t}{2m\overline{v_x}}. \tag{30}$$

Part b

In the text it is shown that,

$$m\overline{v_x^2} = kT. \tag{31}$$

Hence,

$$\overline{v_x} = (\overline{v_x^2})^{1/2} = \sqrt{\frac{kT}{m}} \tag{32}$$

Part c

Equation 30 gives,

$$\frac{\Delta N}{\Delta t} = -\frac{PA}{2m\overline{v_x}}. \tag{33}$$

The minus sign is introduced because ΔN is now the number of molecules that will be *lost* in time Δt through the hole made in the wall. In the limit $\Delta t \to 0$ this can be written as,

$$\frac{dN}{dt} = -\frac{PA}{2m\overline{v_x}}. \tag{34}$$

Using equation 32 and the gas law ($P = NkT/V$) this gives,

$$\frac{dN}{dt} = -\frac{NkTA}{2Vm}\sqrt{\frac{m}{kT}} = -\frac{A}{2V}\sqrt{\frac{kT}{m}}N = -N/\tau, \tag{35}$$

where,

$$\tau = \frac{2V}{A}\sqrt{\frac{m}{kT}}, \tag{36}$$

is called the "characteristic time". Equation 35 can be solved as follows to write N as a function of t.

$$\int_{N(0)}^{N} \frac{dN}{N} = -(1/\tau)\int_0^t dt, \tag{37}$$

where $N(0)$ is the number of molecules in the container at time $t = 0$ and N the number at any time t. Integrating this gives,

$$\ln(N/N(0)) = -t/\tau. \tag{38}$$

Hence,

$$N = N(0)e^{-t/\tau}. \tag{39}$$

Part d

The given values are as follows.

$$A = 1.00\times10^{-6}\ \mathrm{m}^2, \quad V = 1.00\times10^{-3}\ \mathrm{m}^3, \quad k = 1.38\times10^{-23}, \quad T = 300, \quad m = 4.81\times10^{-26}. \tag{40}$$

The above value of m is computed to be the average mass of an air molecule considering air to be made of about 78% Nitrogen, 21% Oxygen and 1% argon. Using these values in equation 36 gives,

$$\tau = 6.82 \text{ sec.} \tag{41}$$

Part e

For a rough calculation, let us use $\tau = 3600$ seconds in equation 36. This is an overestimate as it should take several characteristic times for the tire to deflate completely. Let us also assume the volume inside the tire to be (neglecting tire volume expansion due to pressure) $V = 2.00\times10^{-3}\mathrm{m}^3$. Then,

$$A = \frac{2V}{\tau}\sqrt{\frac{m}{kT}} = 3.8\times10^{-9}\ \mathrm{m}^2 = 3.8\times10^{-3}\ \mathrm{mm}^2. \tag{42}$$

Problem 1.36

Part a

Let P_i (=1 atm) and V_i (=1 L) be the initial pressure and volume and P_f and V_f be the final pressure and volume. Then, using atmosphere as the unit of pressure and liter as the unit of volume,

$$P_f V_f^{\gamma} = P_i V_i^{\gamma} = 1, \tag{43}$$

Hence,

$$V_f = (1/P_f)^{1/\gamma} = (1/7)^{5/7} = 0.25 \text{ L}, \tag{44}$$

as $f = 5$ for diatomic molecules and hence, $\gamma = 7/5$.

Part b

The work done on the gas is,

$$-\int_{V_i}^{V_f} P\,dV = -\int_{V_i}^{V_f} V^{-\gamma}\,dV \tag{45}$$

$$= -\frac{1}{-\gamma+1}(V_f^{-\gamma+1} - V_i^{-\gamma+1}) \tag{46}$$

$$= 1/(\gamma-1)(1/V_f^{\gamma-1} - 1/V_i^{\gamma-1}) = 1.85 \text{ L.atm} = 187 \text{ J}. \tag{47}$$

Part c

If T_i (=300 K) is the initial temperature and T_f is the final temperature,

$$V_f T_f^{5/2} = V_i T_i^{5/2}. \tag{48}$$

As $V_i = 1$L and $V_f = 0.25$L,

$$T_f = T_i V_f^{-2/5} = 300 \times (0.25)^{-2/5} = 522 \text{ K}. \tag{49}$$

Problem 1.37

Once again, we use the formula,

$$V_f T_f^{5/2} = V_i T_i^{5/2}, \tag{50}$$

as, for air, $f = 5$. It is given that $V_f/V_i = 1/20$ and $T_i = 300$K. Hence,

$$T_f = 300 \times 20^{2/5} = 994 \text{ K}. \tag{51}$$

This temperature is high enough to ignite the fuel and hence, no spark plug is needed for that purpose.

Problem 1.39

Part a

For an isothermal process,

$$PV = K_i, \tag{52}$$

where K_i is a constant. Hence,

$$P\,dV + V\,dP = 0, \tag{53}$$

and,

$$\frac{dP}{dV} = -\frac{P}{V}. \tag{54}$$

So, from the definition of bulk modulus, the isothermal bulk modulus is,

$$B_i = -V\frac{dP}{dV} = V\frac{P}{V} = P \tag{55}$$

For an adiabatic process,

$$PV^\gamma = K_a, \tag{56}$$

where K_a is a constant. Hence,

$$\gamma P V^{\gamma-1} dV + V^\gamma dP = 0, \tag{57}$$

and,

$$\frac{dP}{dV} = -\gamma \frac{P}{V}. \tag{58}$$

So, from the definition of bulk modulus, the adiabatic bulk modulus is,

$$B_a = -V\frac{dP}{dV} = V\gamma\frac{P}{V} = \gamma P. \tag{59}$$

Part b

Air being a poor conductor of heat does not easily dissipate the heat generated due to compressions in a sound wave. Hence, quick compressions due to sound waves are expected to adiabatic.

Part c

Using the given expression for sound wave speed,

$$c_s = \sqrt{\frac{B_a}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} \tag{60}$$

Earlier, in equation 22, it was shown that,

$$\rho = \frac{mP}{kT} \tag{61}$$

Hence,

$$c_s = \sqrt{\frac{\gamma P k T}{mP}} = \sqrt{\frac{\gamma k T}{m}}. \tag{62}$$

Earlier the average rms speed of molecules was found to be (equation 32)

$$\overline{v_x} = \sqrt{\frac{kT}{m}}. \tag{63}$$

So, the difference is by a factor of $\sqrt{\gamma}$. Using the average value of $m = 4.81 \times 10^{-26}$ kg for air,

$$c_s = \sqrt{\frac{1.4 \times 1.38 \times 10^{-23} \times 300}{4.81 \times 10^{-26}}} = 347 \text{ m/s}. \tag{64}$$

Part d

The speed of sound does not change directly due to air pressure. So, as long as the temperature does not change, the speed of sound is not expected to change. However, a lower pressure corresponds to lower air density which might cause less heat absorption from the Sun and hence, indirectly change the temperature.

Problem 1.42

Let the specific heat capacity of pasta be c_p and that of water be c_w. Let the initial temperature of water be T_w and that of pasta be T_p. Let the final equilibrium temperature of the two together be T_f. Let the mass of water be m_w and that of pasta be m_p. Now, assuming the process to be quick enough to ignore heat transfer from or to the outside, one may approximate the heat gained by the pasta to be equal to the heat lost by the water. Hence,

$$m_p c_p (T_f - T_p) = m_w c_w (T_w - T_f). \tag{65}$$

So, the final temperature of the mix is,

$$T_f = \frac{m_w c_w T_w + m_p c_p T_p}{m_w c_w + m_p c_p} = \frac{1500 \times 4.2 \times 100 + 340 \times 1.8 \times 25}{1500 \times 4.2 + 340 \times 1.8} = 93^\circ\text{C}. \tag{66}$$

Problem 1.47

It can be assumed that the other ingredients in tea do not change the thermal properties very much from that of water. Let the initial temperature of tea be T_t and that of ice be T_i. Let their final equilibrium temperature be T_f. Let the melting point of ice be T_m. Let the mass of tea be m_t and that of added ice be m_i. Let the specific heat capacity of tea (or water) be c_w and that of ice be c_i. Let the specific latent heat of ice (to water) be l. Then the heat lost by the tea is,

$$Q_t = m_t c_w (T_t - T_f). \tag{67}$$

The heat gained by the ice is the sum of the heats gained in three steps: heating of the ice to its melting point, melting of the ice and heating of the resulting water to the final temperature. Hence, the net heat gained by the ice is,

$$Q_i = m_i c_i (T_m - T_i) + m_i l + m_i c_w (T_f - T_m). \tag{68}$$

Now, assuming the process to be quick enough to ignore heat transfer from or to the outside, $Q_t = Q_i$. Hence,

$$m_t c_w (T_t - T_f) = m_i c_i (T_m - T_i) + m_i l + m_i c_w (T_f - T_m). \tag{69}$$

Solving for m_i gives,

$$m_i = \frac{m_t c_w (T_t - T_f)}{c_i (T_m - T_i) + l + c_w (T_f - T_m)} = \frac{200 \times 1.0 \times (100 - 65)}{0.5 \times (0 - (-15)) + 80 + 1.0 \times (65 - 0)} = 46\text{g}. \tag{70}$$

Problem 1.57

Part a

The R value for the plate glass is,

$$R_g = \frac{3.2 \times 10^{-3}}{0.8} = 4.0 \times 10^{-3} \text{m}^2\text{K/W} \tag{71}$$

The R value for the layer of still air is,

$$R_a = \frac{1.0 \times 10^{-3}}{0.026} = 3.8 \times 10^{-2} \text{m}^2\text{K/W} \tag{72}$$

Part b

Unit conversion gives,

$$\text{m}^2\text{K/W} = (3.28)^2\text{ft}^2 \times 1.8\text{F}^\circ/(3.41\text{BTU/hr}) = 5.68 \text{ ft}^2\text{F}^\circ\text{hr/BTU}. \tag{73}$$

Hence,

$$R_g = 4.0 \times 10^{-3} \times 5.68 = 2.27 \times 10^{-2} \text{ ft}^2\text{F}^\circ\text{hr/BTU}, \tag{74}$$

and,

$$R_a = 3.8 \times 10^{-2} \times 5.68 = 0.216 \text{ ft}^2\text{F}^\circ\text{hr/BTU}, \tag{75}$$

Part c

Using subscripts 1 and 2 for two materials the conduction equation for the two of them can be written as,

$$\frac{Q}{\Delta t} = -k_1 A \frac{\Delta T_1}{\Delta x_1} \text{ and } \frac{Q}{\Delta t} = -k_2 A \frac{\Delta T_2}{\Delta x_2}. \tag{76}$$

As the two materials are placed in layers, they have the same rate of flow $Q/\Delta t$ and area A. Hence, the following quantity is the same for both.

$$\alpha = \frac{Q}{A\Delta t} = -k_1 \frac{\Delta T_1}{\Delta x_1} = -k_2 \frac{\Delta T_2}{\Delta x_2}. = \tag{77}$$

This gives,

$$\Delta T_1 = -\alpha \frac{\Delta x_1}{k_1} \text{ and } \Delta T_2 = -\alpha \frac{\Delta x_2}{k_2}. \tag{78}$$

Hence, using the definition of R value,

$$\Delta T_1 = -\alpha R_1 \text{ and } \Delta T_2 = -\alpha R_2. \tag{79}$$

So, the total temperature difference over the two materials is,

$$\Delta T = \Delta T_1 + \Delta T_2 = -\alpha(R_1 + R_2) = -\alpha R, \tag{80}$$

where, R, the effective value is,

$$R = R_1 + R_2. \tag{81}$$

Now, using the definition of α, the conduction equation can be written as follows.

$$\frac{Q}{\Delta t} = -A\frac{\Delta T}{R}, \tag{82}$$

just as the original individual material equations could be written as,

$$\frac{Q}{\Delta t} = -A\frac{\Delta T_1}{R_1}, \text{ and } \frac{Q}{\Delta t} = -A\frac{\Delta T_2}{R_2}. \tag{83}$$

Part d

The effective R value of the three layers is,

$$R = R_a + R_g + R_a = 3.8 \times 10^{-2} + 4.0 \times 10^{-3} + 3.8 \times 10^{-2} = 0.08 \text{ m}^2\text{K/W}. \tag{84}$$

So, the rate of heat flow is,

$$\frac{Q}{\Delta t} = -\frac{A\Delta T}{R} = -\frac{1 \times (293 - 273)}{.08} = -250 \text{ W}. \tag{85}$$

Problem 62

Consider a small section of the rod extending from x to $x + \Delta x$. Let the amount of heat entering the section at x be $Q(x)$ in a time interval Δt. Let the amount of heat exiting the section at $x+\Delta x$ be $Q(x + \Delta x)$ in the same time interval. From the definition of specific heat capacity we get,

$$\frac{Q(x) - Q(x + \Delta x)}{\Delta t} = mc\frac{\Delta T}{\Delta t} \tag{86}$$

where m is the mass of the section, c the specific heat capacity of the material and ΔT the rise in temperature of the section. From the thermal conductivity equation we get,

$$\frac{Q(x)}{\Delta t} = -k_t A \left.\frac{\partial T}{\partial x}\right|_x, \text{ and } \frac{Q(x + \Delta x)}{\Delta t} = -k_t A \left.\frac{\partial T}{\partial x}\right|_{x+\Delta x}. \tag{87}$$

Here a partial derivative is use because the temperature T is expected to be a function of both x and t. Inserting this in equation 86 we get,

$$k_t A \left(\left.\frac{\partial T}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial T}{\partial x}\right|_x \right) = mc\frac{\Delta T}{\Delta t}. \tag{88}$$

The quantity in parentheses is the change in $\partial T/\partial x$ over a distance of Δx. Hence, one may write the above equation as,

$$k_t A \Delta x \frac{\Delta(\partial T/\partial x)}{\Delta x} = mc\frac{\Delta T}{\Delta t}. \tag{89}$$

In the limit of $\Delta x \to 0$ and $\Delta t \to 0$, this becomes,

$$k_t A\, dx \frac{\partial^2 T}{\partial x^2} = mc\frac{\partial T}{\partial t}. \tag{90}$$

As $A\,dx$ is the volume of the small section and m its mass, $m/A\,dx = \rho$ is the density of the material. Hence, the above equation can be written as,

$$K\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}, \text{ where } K = \frac{k_t}{c\rho}. \tag{91}$$

This is the so-called heat equation in one dimension. The full three dimensional version of it takes the following obvious form.

$$K\nabla^2 T = \frac{\partial T}{\partial t}, \text{ where } \nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}. \tag{92}$$