Solutions
Chapter 8

Problem 3

Let the current be in the x direction and the magnetic field in the z direction. The Hall voltage is measured in the y direction. So the current j_y is zero in the y direction:

$$j_y = epv_{hy} - env_{ey} = 0, \tag{1}$$

where e is the magnitude of electron charge, p the hole density, n the electron density, v_{ey} the electron drift velocity in the y direction and v_{hy} the hole drift velocity in the y direction. So,

$$pv_{hy} = nv_{ey}. \tag{2}$$

The current in the x direction is

$$j_x = (ne\mu_e + pe\mu_h)E_x, \tag{3}$$

where E_x is the x component of the electric field. μ_e and μ_h are the electron and hole mobilities respectively. They are given to be

$$\mu_e = v_{ex}/E_x = e\tau_e/m_e, \tag{4}$$

where v_{ex} is the electron velocity in the x direction, τ_e the electron collision time, m_e the electron effective mass and

$$\mu_h = v_{hx}/E_x = e\tau_h/m_h, \tag{5}$$

where v_{hx} is the hole velocity in the x direction, τ_h the hole collision time and m_h the hole effective mass.

Using equation 6.52 of the textbook for the electron and the hole gives the following 4 equations.

$$\begin{aligned} v_{ex} &= -\mu_e E_x - \omega_{ce}\tau_e v_{ey} && (6) \\ v_{ey} &= -\mu_e E_y + \omega_{ce}\tau_e v_{ex} && (7) \\ v_{hx} &= \mu_h E_x + \omega_{ch}\tau_h v_{hy} && (8) \\ v_{hy} &= \mu_h E_y - \omega_{ch}\tau_h v_{hx}, && (9) \end{aligned}$$

where ω_{ce} and ω_{ch} are the cyclotron frequencies for electrons and holes respectively. Substituting equation 6 in equation 7 for v_{ex} and equation 8 in equation 9 for v_{hx} and then neglecting terms containing $(\omega_{ce}\tau_e)^2$ and $(\omega_{ch}\tau_h)^2$, we get

$$\begin{aligned} v_{ey} &= -\mu_e E_y - \omega_{ce}\tau_e\mu_e E_x && (10) \\ v_{hy} &= \mu_h E_y - \omega_{ch}\tau_h\mu_h E_x && (11) \end{aligned}$$

Dividing equation 10 by equation 11, we get

$$\frac{v_{ey}}{v_{hy}} = \frac{-\mu_e E_y - \omega_{ce}\tau_e\mu_e E_x}{\mu_h E_y - \omega_{ch}\tau_h\mu_h E_x}. \tag{12}$$

However, equation 2 shows that this ratio of velocities is equal to p/n, the ratio of hole to electron densities. Hence, equation 12 becomes

$$\frac{p}{n} = \frac{-\mu_e E_y - \omega_{ce}\tau_e\mu_e E_x}{\mu_h E_y - \omega_{ch}\tau_h\mu_h E_x}. \tag{13}$$

Hence,

$$p\mu_h(E_y - \omega_{ch}\tau_h E_x) = -n\mu_e(E_y + \omega_{ce}\tau_e E_x). \tag{14}$$

Then, collecting all E_x and E_y terms, we get

$$(p\mu_h + n\mu_e)E_y = (p\mu_h\omega_{ch}\tau_h - n\mu_e\omega_{ce}\tau_e)E_x. \tag{15}$$

Then, using equations 4 and 5 as well as the following cyclotron frequency formulas

$$\omega_{ce} = eB/m_e, \quad \omega_{ch} = eB/m_e, \tag{16}$$

we get

$$(p\mu_h + n\mu_e)E_y = (pB\mu_h^2 - nB\mu_e^2)E_x. \tag{17}$$

and

$$\frac{E_y}{E_x} = \frac{pB\mu_h^2 - nB\mu_e^2}{p\mu_h + n\mu_e} \tag{18}$$

Now, from the definition of the Hall coefficient

$$R_H = \frac{E_y}{j_x B}. \tag{19}$$

Using the expression for j_x from equation 3, this gives

$$R_H = \frac{E_y}{(ne\mu_e + pe\mu_h)BE_x}. \tag{20}$$

Then, using the ratio E_y/E_x from equation 18, this gives

$$R_H = \frac{p\mu_h^2 - n\mu_e^2}{e(p\mu_h + n\mu_e)^2} \tag{21}$$

Then, dividing both numerator and denominator by μ_h^2, we get

$$R_H = \frac{p - nb^2}{e(p + nb)^2}, \tag{22}$$

where $b = \mu_e/\mu_h$.

Problem 4

Using the definition of group velocity

$$\vec{\mathbf{v}} = \hbar^{-1}\left(\frac{\partial\epsilon}{\partial k_x}\hat{\mathbf{x}} + \frac{\partial\epsilon}{\partial k_y}\hat{\mathbf{y}} + \frac{\partial\epsilon}{\partial k_z}\hat{\mathbf{z}}\right) \tag{23}$$

Then, using the given expression for ϵ, we get

$$\vec{\mathbf{v}} = \frac{\hbar k_x}{m_t}\hat{\mathbf{x}} + \frac{\hbar k_y}{m_t}\hat{\mathbf{y}} + \frac{\hbar k_z}{m_l}\hat{\mathbf{z}}. \tag{24}$$

Now using equation 8.6 of the textbook, and a magnetic field along the x axis ($\vec{\mathbf{B}} = B\hat{\mathbf{x}}$),

$$\frac{d\vec{\mathbf{k}}}{dt} = -e\left(\frac{k_x}{m_t}\hat{\mathbf{x}} + \frac{k_y}{m_t}\hat{\mathbf{y}} + \frac{k_z}{m_l}\hat{\mathbf{z}}\right) \times (B\hat{\mathbf{x}}) = -e\left(\frac{k_z B}{m_l}\hat{\mathbf{y}} - \frac{k_y B}{m_t}\hat{\mathbf{z}}\right). \tag{25}$$

So, in components,

$$\frac{dk_x}{dt} = 0, \tag{26}$$

$$\frac{dk_y}{dt} = -\frac{ek_z B}{m_l}, \tag{27}$$

$$\frac{dk_z}{dt} = \frac{ek_y B}{m_t}. \tag{28}$$

Taking the derivative of equation 27 and inserting in it dk_z/dt from equation 28, we get

$$\frac{d^2k_y}{dt^2} = -\frac{e^2B^2}{m_l m_t}k_y \tag{29}$$

This gives

$$\frac{d^2k_y}{dt^2} + \omega_c^2 k_y = 0, \tag{30}$$

where

$$\omega_c = \frac{eB}{(m_t m_l)^{1/2}}. \tag{31}$$

This is the cyclotron frequency, as solving equation 30 and then using equation 27 gives

$$k_y = k_0\cos(\omega_c t + \phi), \tag{32}$$

$$k_z = (m_l/m_t)^{1/2}k_0\sin(\omega_c t + \phi). \tag{33}$$

The above equations show that the vector $\vec{\mathbf{k}}$ rotates in an ellipse with the angular velocity ω_c.