Solutions
Chapter 6

Problem 1

First, it is important to note that the variable N has different meanings in equations 17 and 19 of the textbook. In equation 17, N is the total number of electrons in the solid. This N is related to the Fermi energy ϵ_F as shown. However, the N in equation 19 is the number available states between the energies of 0 and ϵ. To avoid confusion we shall call this N_ϵ. So, equation 19 will be,

$$N_\epsilon = \frac{V}{3\pi^2}\left(\frac{2m\epsilon}{\hbar^2}\right)^{3/2}$$

Then the density of states is,

$$D(\epsilon) = \frac{dN_\epsilon}{d\epsilon} = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\epsilon^{1/2}$$

At absolute zero all states are occupied up to the energy ϵ_F. So, the total energy is,

$$U_0 = \int_0^{\epsilon_F} D(\epsilon)\epsilon d\epsilon = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^{\epsilon_F}\epsilon^{3/2}d\epsilon = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\frac{2}{5}\epsilon_F^{5/2}$$

$$= \left[\frac{V}{5\pi^2}\left(\frac{2m\epsilon_F}{\hbar^2}\right)^{3/2}\right]\epsilon_F = \frac{3}{5}N_{\epsilon_F}\epsilon_F = \frac{3}{5}N\epsilon_F.$$

Here it is noted that, by definition, $N_{\epsilon_F} = N$.

Problem 5

The mass of one He^3 atom is (2 protons and 1 neutron),

$$m = 2\times 1.67\times 10^{-27} + 1.67\times 10^{-27} = 5.01\times 10^{-27}\text{ kg}.$$

The density of liquid He^3 is,

$$\rho = 0.081\text{ g cm}^{-3} = 81\text{ kg m}^{-3}.$$

Hence, the number of atoms per unit volume is,

$$N/V = \rho/m = 1.62\times 10^{28}\text{ m}^{-3}.$$

So, the Fermi energy is,

$$\epsilon_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3} = 6.8\times 10^{-23}\text{ J}.$$

The Fermi temperature is,

$$T_F = \epsilon_F/k_B = 4.9\text{ K}.$$

Problem 6

At a frequency ω the electric field can be written as,

$$E = E_0 e^{-i\omega t},$$

and the corresponding drift velocity is,

$$v = v_0 e^{-i\omega t}.$$

Inserting this in the equation,

$$m(dv/dt + v/\tau) = -eE,$$

gives,

$$mv_0(-i\omega + 1/\tau)e^{-i\omega t} = -eE_0 e^{-i\omega t}.$$

Hence,

$$v_0 = \frac{-eE_0}{m(1/\tau - i\omega)} = \frac{-eE_0\tau}{m(1 - i\omega\tau)} = \frac{-eE_0\tau(1 + i\omega\tau)}{m(1 + (\omega\tau)^2)}.$$

In the last step, the numerator and denominator are both multiplied by $(1 + i\omega\tau)$ to make the denominator real. Then, the current density is,

$$j = -nev = -nev_0 e^{-i\omega t} = \frac{ne^2 E_0\tau(1 + i\omega\tau)}{m(1 + (\omega\tau)^2)} e^{-i\omega t} = \frac{ne^2 E\tau(1 + i\omega\tau)}{m(1 + (\omega\tau)^2)}.$$

Then, the conductivity is,

$$\sigma(\omega) = j/E = \frac{ne^2\tau(1 + i\omega\tau)}{m(1 + (\omega\tau)^2)} = \frac{ne^2\tau}{m}\left(\frac{1 + i\omega\tau}{1 + (\omega\tau)^2}\right) = \sigma(0)\left(\frac{1 + i\omega\tau}{1 + (\omega\tau)^2}\right),$$

where,

$$\sigma(0) = \frac{ne^2\tau}{m}.$$

Problem 9

Equation 52 of the textbook gives,

$$v_x = -\frac{e\tau}{m}E_x - \omega_c\tau v_y; \quad v_y = -\frac{e\tau}{m}E_y + \omega_c\tau v_x; \quad v_z = -\frac{e\tau}{m}E_z.$$

Substituting v_y from the second equation into the first, and v_x from the first equation into the second gives,

$$\begin{aligned} v_x &= -\frac{e\tau}{m}E_x - \omega_c\tau(-\frac{e\tau}{m}E_y + \omega_c\tau v_x), \\ v_y &= -\frac{e\tau}{m}E_y + \omega_c\tau(-\frac{e\tau}{m}E_x - \omega_c\tau v_y). \end{aligned}$$

Solving for v_x and v_y from this and including the v_z equation, produces the following three equations,

$$\begin{aligned} v_x &= (1+(\omega_c\tau)^2)^{-1}\left(-\frac{e\tau}{m}E_x + \frac{e\omega_c\tau^2}{m}E_y\right), \\ v_y &= (1+(\omega_c\tau)^2)^{-1}\left(-\frac{e\omega_c\tau^2}{m}E_x - \frac{e\tau}{m}E_y\right), \\ v_z &= -\frac{e\tau}{m}E_z. \end{aligned}$$

Written in matrix notation, this looks as follows.

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \frac{-e\tau}{m(1+(\omega_c\tau)^2)}\begin{pmatrix} 1 & -\omega_c\tau & 0 \\ \omega_c\tau & 1 & 0 \\ 0 & 0 & (1+(\omega_c\tau)^2) \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}.$$

As the current density is $\vec{\mathbf{j}} = -ne\vec{\mathbf{v}}$, multiplying the above equation by $-ne$ and noting that $\sigma_0 = ne^2\tau/m$, gives the following.

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \frac{\sigma_0}{1+(\omega_c\tau)^2}\begin{pmatrix} 1 & -\omega_c\tau & 0 \\ \omega_c\tau & 1 & 0 \\ 0 & 0 & (1+(\omega_c\tau)^2) \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}.$$