Solutions
Chapter 5

Problem 1

Part a

The one dimensional density of states is given by,

$$D(\omega) = \frac{L}{\pi}\frac{1}{d\omega/dK}.$$

For the monatomic lattice,

$$\omega = (4C/M)^{1/2}\sin(Ka/2) = \omega_m \sin(Ka/2),$$

where,

$$\omega_m = (4C/M)^{1/2},$$

is the maximum frequency. Hence,

$$d\omega/dK = \omega_m(a/2)\cos(Ka/2) = \omega_m(a/2)(1-\sin^2(Ka/2))^{1/2} = (a/2)(\omega_m^2 - \omega_m^2\sin^2(Ka/2))^{1/2}$$
$$= (a/2)(\omega_m^2 - \omega^2)^{1/2}.$$

Substituting this in the expression for $D(\omega)$ gives,

$$D(\omega) = \frac{2L}{\pi a}\frac{1}{(\omega_m^2-\omega^2)^{1/2}} = \frac{2N}{\pi}\frac{1}{(\omega_m^2-\omega^2)^{1/2}},$$

as $N = L/a$.

Part b

In three dimensions,

$$D(\omega) = \frac{VK^2}{2\pi^2}\frac{1}{d\omega/dK}.$$

If

$$\omega = \omega_0 - AK^2,$$
$$\frac{d\omega}{dK} = -2AK.$$

Hence,

$$D(\omega) = -\frac{VK}{4\pi^2 A}$$

As

$$K = \pm A^{-1/2}(\omega_0 - \omega)^{1/2},$$

$$D(\omega) = \frac{V}{4\pi^2 A^{3/2}}(\omega_0 - \omega)^{1/2}.$$

Here the negative sign for K is chosen to obtain a positive $D(\omega)$.

As $\omega > \omega_0$ is not allowed by the dispersion relation, $D(\omega) = 0$ for such a condition.