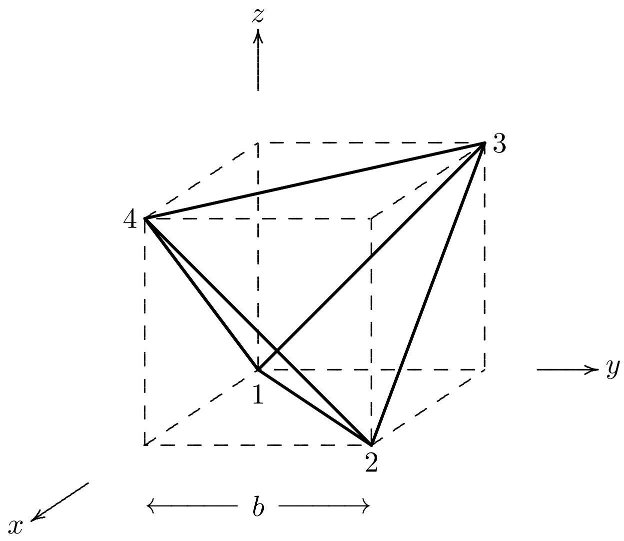
Solutions
Chapter 1

Problem 1



Consider the tetrahedron (1234). The perpendicular direction to the plane (234) is given by the cross product of vectors along two of its sides:

$$\vec{\mathbf{r}}_{23} = b\hat{\mathbf{x}} - b\hat{\mathbf{z}}, \quad \vec{\mathbf{r}}_{34} = -b\hat{\mathbf{x}} + b\hat{\mathbf{y}}.$$

Then the cross product is

$$\vec{\mathbf{r}}_{234} = \vec{\mathbf{r}}_{23} \times \vec{\mathbf{r}}_{34} = b^2\hat{\mathbf{x}} + b^2\hat{\mathbf{y}} + b^2\hat{\mathbf{z}}.$$

Then the unit vector in this direction can be written as

$$\hat{\mathbf{r}}_1 = \frac{\vec{\mathbf{r}}_{234}}{|\vec{\mathbf{r}}_{234}|} = \frac{\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}}{\sqrt{3}}.$$

Similarly, the unit vector perpendicular to the plane (123) is written as

$$\hat{\mathbf{r}}_4 = \frac{\vec{\mathbf{r}}_{123}}{|\vec{\mathbf{r}}_{123}|} = \frac{-\hat{\mathbf{x}} + \hat{\mathbf{y}} - \hat{\mathbf{z}}}{\sqrt{3}}.$$

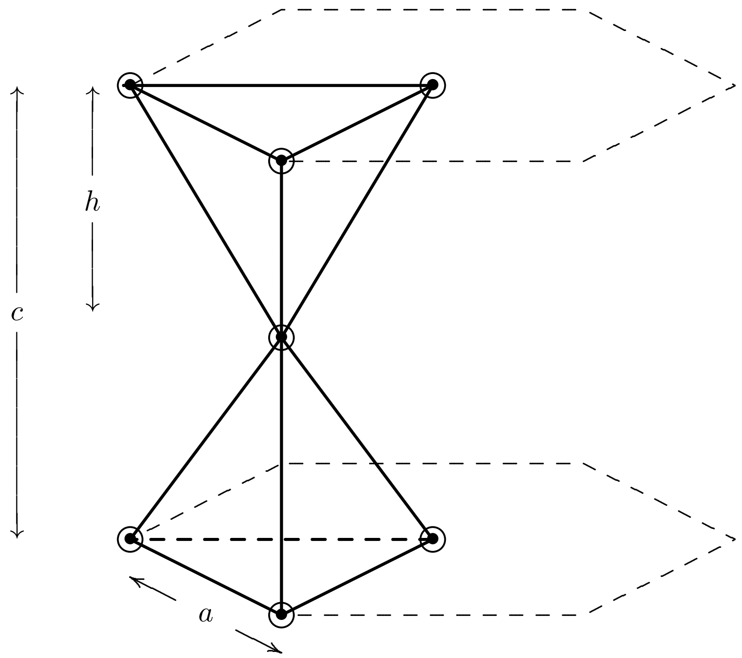
The scalar (dot) product of $\hat{\mathbf{r}}_1$ and $\hat{\mathbf{r}}_4$ gives the cosine of the angle θ between them:

$$\cos\theta = \hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_4 = -1/3.$$

Hence,

$$\theta = 109.47^\circ.$$

Problem 3



The figure above shows how the height c of the hexagonal cell is twice the height h of two tetrahedrons of edges a. In problem 1, the unit vector perpendicular to the face (234) was found to $\hat{\mathbf{r}}_1$. h is the projection of one of three edges along this direction. Let us choose the edge given by the following vector.

$$\vec{\mathbf{r}}_{12} = b\hat{\mathbf{x}} + b\hat{\mathbf{y}}.$$

Then,

$$h = \vec{\mathbf{r}}_{12} \cdot \hat{\mathbf{r}}_1 = 2b/\sqrt{3}.$$

As a is the length of one of the edges of the tetrahedron, $a = \sqrt{2}b$. Using this in the above expression for h gives,

$$h = \frac{\sqrt{2}a}{\sqrt{3}}.$$

Then,

$$c = 2h = \frac{2\sqrt{2}a}{\sqrt{3}}.$$

And,

$$\frac{c}{a} = \sqrt{\frac{8}{3}} = 1.633.$$