Solutions

Chapter 3

Problem 1

$$\lambda_m T = 2.898 \times 10^{-3}.$$

Hence,

$$\lambda_m = 2.898 \times 10^{-3}/300 = 9.66 \times 10^{-6} \text{ m}$$

This is significantly higher than the visible range. It is in the far infra-red range.

Problem 2

$$\lambda_m T = 2.898 \times 10^{-3}.$$

Hence,

$$T = 2.898 \times 10^{-3}/3.55 \times 10^{-6} = 816 \text{ K}$$

Problem 5

For small h the following first order approximation for an exponential can be used.

$$e^{h\nu/kT} \simeq 1 + h\nu/kT$$

Then the expression for $S(\nu)$ can be written as

$$S(\nu) = \frac{2\pi h\nu^3}{c^2}\frac{1}{1 + h\nu/kT - 1} = \frac{2\pi h\nu^3}{c^2}\frac{1}{h\nu/kT} = \frac{2\pi\nu^2 kT}{c^2}$$

This is the Rayleigh-Jeans result.

Problem 6

For long wavelengths, frequency ν is small and hence the first order approximation for an exponential used in the last problem can be used again giving the same result.

Problem 7

The expression for $\overline{E}$ is

$$\overline{E} = \frac{\sum_n E_n N(E_n)}{\sum_n N(E_n)}$$

where $E_n = nh\nu^2$ and hence,

$$N(E_n) = N(0)e^{-nh\nu^2/kT}$$

Using $x = e^{-h\nu^2/kT}$, the sum in the denominator for the $\overline{E}$ expression can be written as

$$\sum_n N(E_n) = N(0)\sum_n x^n = N(0)\frac{1}{1-x}$$

The sum in the numerator is

$$\sum_n E_n N(E_n) = N(0)h\nu^2 \sum_n nx^n = N(0)h\nu^2 \frac{x}{(1-x)^2}$$

Hence,

$$\overline{E} = h\nu^2 \frac{x}{1-x} = h\nu^2 \frac{1}{e^{h\nu^2/kT} - 1}$$

So,

$$S(\nu) = \frac{2\pi\nu^2}{c^2}\overline{E} = \frac{2\pi h\nu^4}{c^2}\frac{1}{e^{h\nu^2/kT} - 1}$$

Then,

$$R(\lambda) = \frac{c}{\lambda^2}S(c/\lambda) = \frac{2\pi hc^3}{\lambda^6}\frac{1}{e^{hc^2/\lambda^2 kT} - 1}$$

Problem 8

$$R(\lambda) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}$$

Hence,

$$e^{hc/\lambda kT} = 1 + \frac{2\pi hc^2}{\lambda^5 R(\lambda)}$$

and

$$\frac{hc}{\lambda kT} = \ln\left(1 + \frac{2\pi hc^2}{\lambda^5 R(\lambda)}\right)$$

Then

$$T = \frac{hc}{\lambda k}\left[\ln\left(1 + \frac{2\pi hc^2}{\lambda^5 R(\lambda)}\right)\right]^{-1} = 1100\ \mathrm{K}$$

Problem 9

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta) = 4.15 \times 10^{-12}\text{ m}$$

Then

$$\lambda' = \lambda + 4.15 \times 10^{-12} = 7.41 \times 10^{-11}\text{ m}$$

Problem 11

$$\lambda = h/p = \frac{h}{mv} = 6.07 \times 10^{-5}\text{ m}$$

Problem 12

$$\lambda = h/p = \frac{h}{mv} = 1.84 \times 10^{-34}\text{ m}$$

Problem 14

The momentum of the particle is (using the mass-shell condition)

$$p = c^{-1}\sqrt{E^2 - m_0^2 c^4}$$

where the total energy E is the sum of the rest energy $m_0 c^2$ and the kinetic energy K:

$$E = m_0 c^2 + K.$$

As the kinetic energy is provided completely by the electric potential, $K = eV$. Hence,

$$p = c^{-1}\sqrt{(m_0 c^2 + eV)^2 - m_0^2 c^4}$$

Then,

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{(m_0 c^2 + eV)^2 - m_0^2 c^4}}$$

For $eV \ll m_0 c^2$, (using the approximation $(1 + x)^n \simeq 1 + nx$ for $x \ll 1$)

$$(m_0 c^2 + eV)^2 = m_0^2 c^4 \left(1 + \frac{eV}{m_0 c^2}\right)^2 \simeq m_0^2 c^4 \left(1 + \frac{2eV}{m_0 c^2}\right) = m_0^2 c^4 + 2eV m_0 c^2$$

Replacing this in the equation for λ gives

$$\lambda \simeq \frac{hc}{\sqrt{2eV m_0 c^2}} = \frac{h}{\sqrt{2eV m_0}}$$

Problem 15

The momentum of the electron is

$$p = \frac{h}{\lambda} = 9.47 \times 10^{-24} \text{ kg.m/s}$$

The relativistic momentum formula is

$$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

Hence,

$$p^2 = \frac{m_0^2 v^2}{1 - v^2/c^2}$$

and

$$p^2 - p^2 v^2/c^2 = m_0^2 v^2$$

Then

$$v = \frac{p}{\sqrt{m_0^2 + p^2/c^2}} = 1.04 \times 10^7 \text{ m/s}$$

The non-relativistic formula for p can be seen to give the same result upto three significant figures.

Problem 16

For minimum possible Δp_x

$$\Delta p_x \Delta x = \frac{h}{4\pi}$$

Hence,

$$\Delta p_x = \frac{h}{4\pi \Delta x} = \frac{6.63 \times 10^{-34}}{4\pi \times 2.0 \times 10^{-2}} = 2.64 \times 10^{-33} \text{ kg.m/s}$$

Similarly,

$$\Delta p_y = \frac{h}{4\pi \Delta y} = \frac{6.63 \times 10^{-34}}{4\pi \times 2.0 \times 10^{-3}} = 2.64 \times 10^{-32} \text{ kg.m/s}$$