# Voltage Sources<sup>\*</sup>

## Object

To measure the emfs and the internal resistances of given chemical cells.

### Theory

The emf of a cell is difficult to measure because a voltmeter connected across it will always draw some current. This current produces a potential drop across the internal resistance of the cell. As a result, the voltmeter reading becomes less than the emf. A spent cell has an increased internal resistance which lowers its voltage as measured by a voltmeter. However, the emf should never change as it is a property peculiar to the chemical reaction producing it.

Let us consider a cell of emf  $\mathcal{E}$  and internal resistance r connected to a voltmeter of internal resistance R as shown below. Let the voltmeter reading be V.



If i is the current in the loop, then Kirchhoff's loop rule gives

$$\mathcal{E} - ir - iR = 0.$$

The voltage across the cell, as measured by the voltmeter can be written as

$$V = \mathcal{E} - ir.$$

Eliminating i from these two equations, we get

$$r = \frac{(\mathcal{E} - V)R}{V}.$$
(1)

The measured voltage V and the internal resistance of the voltmeter are known quantities. That still leaves r and  $\mathcal{E}$  as unknowns in this equation. To find them, we need another equation. This we can do by connecting a known resistance  $R_b$  in parallel to the voltmeter which lowers the effective voltmeter resistance to the parallel combination of R and  $R_b$ :

$$R_1 = \frac{RR_b}{R + R_b}.$$
(2)

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As a result the voltmeter reading will change. Let the new voltmeter reading be  $V_1$ . So, in equation 1, we can replace R by  $R_1$  and V by  $V_1$  to get

$$r = \frac{(\mathcal{E} - V_1)R_1}{V_1}.$$
 (3)

Now, we can eliminate r between equations 1 and 3 to get

$$\mathcal{E} = \frac{R - R_1}{R/V - R_1/V_1}.$$
 (4)

This allows us to compute the emf  $\mathcal{E}$  of the cell from measured quantities. We can use this computed emf in equation 1 or 3 to find r.

#### The measurement method

Connect the voltmeter directly across a cell to measure its voltage V. Read the internal resistance R of the voltmeter stamped on its face. Then, connect a resistance box in parallel to the voltmeter. Start with a high resistance  $R_b$  for the resistance box. Then lower  $R_b$  until the voltmeter reading is significantly less than V (maybe about half of V). Measure this voltage  $V_1$ . Compute  $R_1$  using equation 2. Then compute  $\mathcal{E}$  from equation 4 and r from equation 1 or 3. We shall call these the measured values of  $\mathcal{E}$  and r.

#### Some trials

Make the above measurements for two separate cells (say  $\mathcal{E}$  and r for the first cell and  $\mathcal{E}_1$  and  $r_1$  for the second cell).

Combine the two tested cells in parallel and measure the emf  $\mathcal{E}_p$  and internal resistance  $r_p$  of the combination. Compute  $\mathcal{E}_p$  and  $r_p$  theoretically from the known values of  $\mathcal{E}$ ,  $\mathcal{E}_1$ , r and  $r_1$ . Compare these theoretical values to the measured values.

Combine the two tested cells in series and measure the emf  $\mathcal{E}_s$  and internal resistance  $r_s$  of the combination. Compute  $\mathcal{E}_s$  and  $r_s$  theoretically from the known values of  $\mathcal{E}$ ,  $\mathcal{E}_1$ , r and  $r_1$ . Compare these theoretical values to the measured values.