Solutions
Chapter 10

Problem 1

The flux is

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = B\cos\theta \int dA = BA\cos\theta$$

To obtain the maximum emf we need the maximum value of $\cos\theta$ which is 1 (for $\theta = 0$). Then the maximum emf is pruduced by the maximum rate of change of flux:

$$\mathcal{E} = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi(0.10/2)^2 \times 0.20 = 1.6 \times 10^{-3} \text{ V}$$

Problem 2

Let us choose the direction of the loop surface element to be the same as that of the magnetic field. Then the induced emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA) = -A\frac{dB}{dt}$$

The induced current is

$$i = \frac{\mathcal{E}}{R} = -\frac{A}{R}\frac{dB}{dt} = -\frac{\pi(0.10)^2}{5.0}\frac{dB}{dt} = -(6.3 \times 10^{-3})\frac{dB}{dt}$$

Part a

In this range $dB/dt = 1.0\text{T/s}$. Hence,

$$i = -6.3 \times 10^{-3} \times 1.0 = -6.3 \times 10^{-3} \text{ A}$$

Part b

In this range $dB/dt = -0.50\text{T/s}$. Hence,

$$i = -6.3 \times 10^{-3} \times (-0.50) = 3.1 \times 10^{-3} \text{ A}$$

Part c

In this range $dB/dt = 0.0\text{T/s}$. Hence,

$$i = -6.3 \times 10^{-3} \times 0.0 = 0.0 \text{ A}$$

Part d

In this range $dB/dt = -0.50\text{T/s}$. Hence,

$$i = -6.3 \times 10^{-3} \times (-0.50) = 3.1 \times 10^{-3} \text{ A}$$

Part e

In this range $dB/dt = 0.0\text{T/s}$. Hence,

$$i = -6.3 \times 10^{-3} \times 0.0 = 0.0 \text{ A}$$

Problem 3

Induced emf is

$$\mathcal{E} = -\frac{d}{dt}(NBA)$$

Here A is the area of cross-section of the solenoid as the magnetic field is restricted within it. So

$$\mathcal{E} = -NA\frac{dB}{dt} = -NA\frac{d}{dt}(\mu_0 n i_s)$$

where n is the turn density of the solenoid and i_s is the current in it. So

$$\mathcal{E} = -NA\mu_0 n\frac{di_s}{dt}$$

So the current in the coil is

$$i = \frac{\mathcal{E}}{R} = -\frac{NA\mu_0 n}{R}\frac{di_s}{dt} = -\frac{20 \times \pi(0.025)^2 \times 4\pi \times 10^{-7} \times 100}{8.0} \times 1.5 = -9.3 \times 10^{-7} \text{ A}$$

Problem 4

Part a

The induced emf is:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

But $\mathcal{E} = iR$, and $i = dq/dt$. Hence,

$$R\frac{dq}{dt} = -\frac{d\Phi_B}{dt}$$

This gives:

$$Rdq = -d\Phi_B$$

Integrating this from time zero to t, gives:

$$R\int_0^{q(t)} dq = -\int_{\Phi_B(0)}^{\Phi_B(t)} d\Phi_B$$

which is

$$Rq(t) = \Phi_B(0) - \Phi_B(t).$$

Part b

Choosing the initial direction of the area element to be along the magnetic field gives

$$\Phi_B(0) = NBA\cos(0) = NBA.$$

Then the final flux is

$$\Phi_B(t) = NBA\cos(180^\circ) = -NBA.$$

So,

$$Rq(t) = \Phi_B(0) - \Phi_B(t) = 2NBA$$

So,

$$B = \frac{Rq(t)}{2NA} = \frac{15 \times 5.5 \times 10^{-3}}{2 \times 50 \times 2.0 \times 10^{-4}} = 4.1 \text{ T}.$$

Problem 5

The induced emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(N\vec{\mathbf{B}} \cdot \vec{\mathbf{A}}) = -\frac{d}{dt}(NBA\cos\theta)$$

where θ is the angle between $\vec{\mathbf{B}}$ and the area $\vec{\mathbf{A}}$. So

$$\mathcal{E} = -NBA\frac{d}{dt}(\cos\theta)$$

As frequency is f, $\theta = \omega t = 2\pi f t$. So

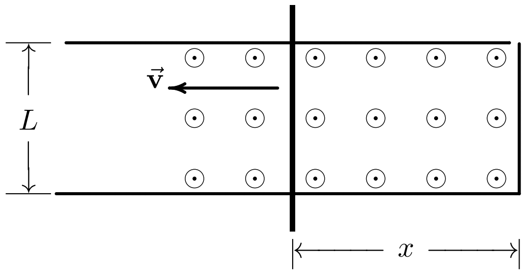
$$\mathcal{E} = -NBA\frac{d}{dt}(\cos(2\pi f t)) = 2\pi f NBA\sin(2\pi f t)$$

The area $A = ab$. So

$$\mathcal{E} = 2\pi f NabB\sin(2\pi f t) = \mathcal{E}_0\sin(2\pi f t)$$

where $\mathcal{E}_0 = 2\pi f NabB$.

Problem 6



Part a

The induced emf in the closed loop created by the rod and the rails is also the emf across the rod alone as the rails have negligible resistance. This emf is

$$\mathcal{E} = -\frac{d}{dt}\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = -\frac{d}{dt}(BA) = -B\frac{d}{dt}(Lx) = -BL\frac{dx}{dt} = -BLv = -0.10 \text{ V}$$

Here $d\vec{\mathbf{A}}$ is chosen to be directed out of the page and hence positive emf is counterclockwise. The negative result means that the emf is clockwise.

Part b

The rails have negligible resistance. So the total resistance of the loop is 20Ω. So the current is:

$$i = \frac{\mathcal{E}}{R} = \frac{-0.10}{20} = -5.0 \times 10^{-3} \text{ A}$$

The negative sign means that the current is clockwise and hence, upwards along the rod.

Part c

The rate of heat loss in the rod is

$$P = i^2 R = (5.0 \times 10^{-3})^2 \times 20 = 5.0 \times 10^{-4} \text{ W}$$

Problem 7

Part a

For the given direction of the loop, the direction of $d\vec{\mathbf{A}}$ is into the page (right-hand-rule). So, from Faraday's law

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = -\frac{d}{dt}\int B\, dA(+1) = -\frac{d}{dt}(BA) = -A\frac{dB}{dt} = -(2.0)^2 \times (-2.5) = 10 \text{ V}$$

Part b

For the given direction of the loop, the direction of $d\vec{\mathbf{A}}$ is out of the page (right-hand-rule). So, from Faraday's law

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = -\frac{d}{dt}\int B\,dA(+1) = -\frac{d}{dt}(BA) = -A\frac{dB}{dt} = -(1.5)^2 \times (4.0) = -9.0 \text{ V}$$

Part c

Once again $d\vec{\mathbf{A}}$ is out of the page. Now the integral over area needs to be separated for the two regions of magnetic field which are labeled L and R for "left" and "right". Hence,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = -\frac{d}{dt}\left[\int_L \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} + \int_R \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}\right] = -\frac{d}{dt}\left[\int_L B\,dA(-1) + \int_R B\,dA(+1)\right] =$$

$$= -\frac{d}{dt}[-B_L A_L + B_R A_R] = A_L\frac{dB_L}{dt} - A_R\frac{dB_R}{dt} = [(2.0)^2 \times (-2.5) - (1.5)^2 \times (4.0)] = -19 \text{ V}.$$

Problem 8

Part a

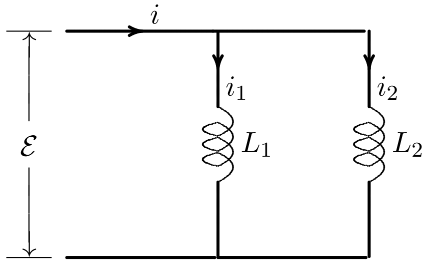
$$\mathcal{E}_L = -L\frac{di}{dt}$$

As $\mathcal{E}_L$ and i are in the same direction, di/dt must be negative to make $\mathcal{E}_L$ positive. So i is decreasing.

Part b

$$L = \frac{-\mathcal{E}_L}{di/dt} = \frac{-25}{-42} = 0.60 \text{ H}$$

Problem 9



From Kirchhoff's current rule:

$$i = i_1 + i_2 \Longleftrightarrow \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}.$$

$\mathcal{E}$ being the same across both inductors:

$$\mathcal{E} = -L_1 \frac{di_1}{dt}, \;\; \mathcal{E} = -L_2 \frac{di_2}{dt}, \;\; \mathcal{E} = -L_{\text{eq}} \frac{di}{dt}$$

where L_{eq} is the equivalent inductance of the two inductors. The equations above can be combined to give:

$$\frac{\mathcal{E}}{L_{\text{eq}}} = \frac{\mathcal{E}}{L_1} + \frac{\mathcal{E}}{L_2}$$

Hence,

$$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}$$

Problem 10

At $t = 0$, $i = i_0 = 2.0$A. At any other time t

$$i = i_0 e^{-tR/L}$$

or

$$\ln\left(\frac{i}{i_0}\right) = -tR/L$$

Solving for L gives:

$$L = -\frac{Rt}{\ln(i/i_0)} = \frac{Rt}{\ln(i_0/i)} = \frac{5.0 \times 0.010}{\ln(2.0/0.50)} = 0.036 \text{ H}$$

Problem 11

Part a

The current rises as follows.

$$i = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L})$$

Hence,

$$e^{-Rt/L} = 1 - \frac{iR}{\mathcal{E}}$$

Then,

$$-\frac{Rt}{L} = \ln\left(1 - \frac{iR}{\mathcal{E}}\right)$$

So,

$$L = \frac{-Rt}{\ln(1 - iR/\mathcal{E})} = 6.3 \text{ H}$$

Part b

The stored energy is

$$U = Li^2/2 = 6.3 \times (1.5)^2/2 = 7.1 \text{ J}$$