Experimental Error*

Object

To study error estimation in experimental measurements using average speed measurement as an example.

Theory

Statistical analysis of experimental data can be quite elaborate. However, here we shall consider just the estimation of an average value and an experimental error. As an example, we shall measure the average speed of a toy car going down a slope. The formula for average speed is

$$v_a = x/t, \tag{1}$$

where x is the distance travelled and t is the time taken to do so. The average of n values of v_a is written as

$$\overline{v}_a = \frac{1}{n}\sum_{i=1}^{n} v_{ai}, \tag{2}$$

where v_{ai} is the i^{th} value of v_a. The error is estimated by the standard deviation:

$$\sigma_v = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(v_{ai} - \overline{v}_a)^2}, \tag{3}$$

One may also estimate the averages and standard deviations of the directly measured quantities x and t (using formulas similar to equations 2 and 3). They can be denoted as $\overline{x}$, σ_x, $\overline{t}$ and σ_t. If the errors are small compared to the averages, the following relationship is approximately true.

$$\sigma_v = \frac{\overline{x}\sigma_t + \overline{t}\sigma_x}{\overline{t}^2}. \tag{4}$$

See if you can derive this relationship. Note the similarity of this to the formula for the derivative of a quotient.

In the present example the computed quantity is a quotient of the measured quantities (equation 1). Hence, we have the relationship of errors as given by equation 4. If the computed quantity were the product of the measured quantities the relationship of the errors would be different. As an example, if we were to measure v_a directly by some other method and then computed x from measured quantities as follows,

$$x = v_a t, \tag{5}$$

then the relationship of the errors would be

$$\sigma_x = \overline{v}_a \sigma_t + \overline{t} \sigma_v. \tag{6}$$

See if you can derive this relationship. Note the similarity of this to the formula for the derivative of a product.

The measurement method

A toy car is released from rest at the top of a sloping surface. The time taken (t) for it to reach the bottom of the slope is measured with a stop watch. The length (x) of the sloping surface is measured after every run. Although x is expected to be the same every time, you might still find some variation. Then v_a is computed for each run.

Some trials

Make a table for the measured quantities (x and t) and the computed quantity (v_a). Then, make repeated measurements of x and t until you are bored silly. Compute v_a for each run. Finally, compute averages and errors (standard deviations) for x, t and v_a. See if you can verify equation 4.

Think of other experiments you can do with the same apparatus where error estimation would be important.