Small Oscillations*

Object

To study small oscillations of springs and pendulums and measure the acceleration due to gravity.

Theory

An object in stable equilibrium usually has a tendency to oscillate about the position of equilibrium. This is because a small enough displacement from the position of equilibrium produces a restoring force in the opposite direction. For the lowest order of approximation, this restoring force is proportional to the displacement. So, for one-dimensional motion, the restoring force is given by

$$F = -kx, \tag{1}$$

where x is the displacement from the equlibrium position and k is a constant.

A standard example of such an oscillating object is a mass suspended at the end of a spring. Another example is a pendulum. For a spring, k is called the spring constant or the stiffness constant. For a pendulum made of a small size mass suspended at the end of a light string, it can be shown that

$$k = mg/l, \tag{2}$$

where m is the mass, g is the acceleration due to gravity and l is the length of the string. Equation 2 is true as long as x is much smaller than l.

If all forces other than the restoring force are negligible, then Newton's second law gives

$$m\frac{d^2x}{dt^2} = -kx, \tag{3}$$

where t is time. The general solution of this equation is

$$x = x_m \sin(2\pi t/T + \phi), \tag{4}$$

where x_m and ϕ are constants that depend on initial conditions and T is the time period of one complete oscillation. It can be seen that

$$T = 2\pi\sqrt{m/k}. \tag{5}$$

*©Tarun Biswas (2000)

For the case of the pendulum, using equations 2 and 5 gives

$$T = 2\pi\sqrt{l/g}. \tag{6}$$

The measurement method

Suspend a spring vertically and hang a known mass m from it. Measure the extension of the spring in equilibrium. As k is a constant, this measurement can be used to find k using equation 1 and the fact that $F = mg$. Using the same mass or a different one start vertical oscillations of the spring. Measure the time period T of oscillations by timing 20 oscillations using a stop watch and dividing by 20. Compare this measured value of T to the computed value using equation 5 and the measured value of k (find the percentage difference).

For the pendulum, measure T the same way as above and then use equation 6 to find g the acceleration due to gravity. Compare this value of g to the one measured directly in a previous experiment using a air-track (find the percentage difference).

Some trials

Measure the spring time period for several different masses and compare with the computed value. See if the value of the mass affects the agreement of the two. Does the amplitude of oscillations make any difference?

Find g using the pendulum for various different starting amplitudes. What is the effect of amplitude on the measured value of g?