Rigid Body Equilibrium*

Object

To verify the rigid body equilibrium conditions.

Theory

If a rigid rod is in equlibrium under the influence of three forces in a single vertical plane, the following three equations are true.

$$F_{1x} + F_{2x} + F_{3x} = 0, \quad (1)$$
$$F_{1y} + F_{2y} + F_{3y} = 0, \quad (2)$$
$$\tau_{1z} + \tau_{2z} + \tau_{3z} = 0, \quad (3)$$

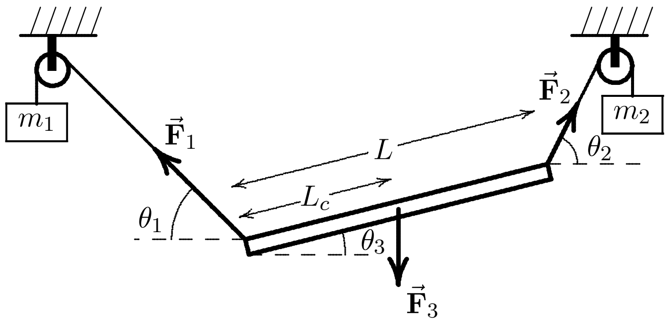
where F_{ix} denotes the x components of the forces, F_{iy} the y components and τ_{iz} the z components of the torques. The forces are in the x-y plane with the y-axis vertical. The forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ are applied at the ends of the rod at angles θ_1 and θ_2 to the horizontal (as shown in the figure). The rod, in equilibrium, makes an angle of θ_3 to the horizontal. $\vec{\mathbf{F}}_3$ is the weight of the rod. Using these conditions, equations 1, 2 and 3 reduce to the following.

$$F_1 \cos\theta_1 = F_2 \cos\theta_2, \quad (4)$$
$$F_1 \sin\theta_1 + F_2 \sin\theta_2 = F_3, \quad (5)$$
$$LF_2 \sin(\theta_2 - \theta_3) = L_c F_3 \cos\theta_3, \quad (6)$$

where the point of application of $\vec{\mathbf{F}}_1$ is chosen as the origin and the points of application of $\vec{\mathbf{F}}_2$ and $\vec{\mathbf{F}}_3$ are a distance L and L_c away from the origin. Note that the point of application of $\vec{\mathbf{F}}_3$ is the center of mass of the rod.

The measurement method

The setup is shown below. A rigid rod is suspended by two strings. Each string goes over a pulley with a mass hung at its other end.



As $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ are the string tensions, their magnitudes are $m_1 g$ and $m_2 g$ respectively. If the mass of the rod is M then the magnitude of $\vec{\mathbf{F}}_3$ is Mg.

Initially, measure the mass M of the rod and the lengths L and L_c. You may find the position of the center of mass by suspending it by a single string and shifting the point of suspension until the rod is approximately horizontal.

Next set up the rod as shown in the figure. You may use any combination of masses m_1 and m_2. Measure the angles θ_1, θ_2 and θ_3. Now you have all quantities appearing in equations 4, 5 and 6. Compare the left and right sides of each equation by finding the percentage difference. A small percentage difference means good agreement of theory and experiment.

Some trials

Make measurements with different sets of masses m_1 and m_2 and test the theory in each case. Display your results in a table.

What would be the effect of shifting the points where the strings are attached to the rod?