Force Equilibrium*

Object

To verify the force equilibrium condition.

Theory

If a point object is in equilibrium (stationary), the total force acting on it must be zero. If there are three different forces ($\vec{\mathbf{T}}_1$, $\vec{\mathbf{T}}_2$ and $\vec{\mathbf{T}}_3$) acting on such an object we may conclude:

$$\vec{\mathbf{T}}_1 + \vec{\mathbf{T}}_2 + \vec{\mathbf{T}}_3 = 0. \quad (1)$$

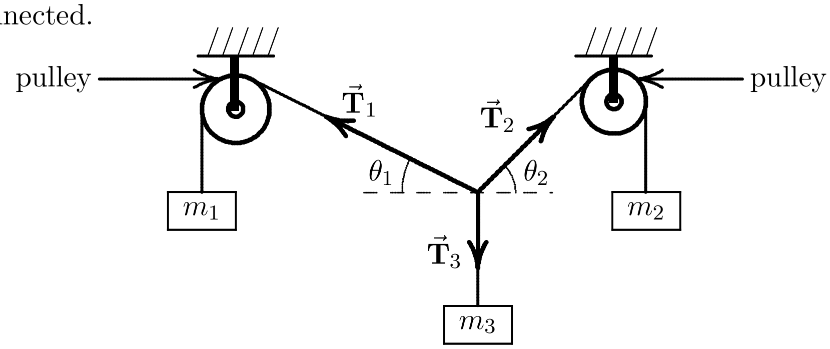
using the concept of vector addition. In two dimensions one may write the above equation in component form as the following two equations.

$$T_{1x} + T_{2x} + T_{3x} = 0, \quad (2)$$
$$T_{1y} + T_{2y} + T_{3y} = 0, \quad (3)$$

where the subscripts x and y denote the corresponding vector components. So, to test the force equilibrium condition, we need to test equations 2 and 3.

The measurement method

The setup is shown below. The object at equilibrium is the point where the three strings are connected.



The tensions in the three strings coming out of the junction are the three vectors. The two strings that go over the pulleys have magnitudes of their tensions (T_1 and T_2) equal to the weights hanging over the pulleys (m_1g and m_2g). This is assuming the pulleys to be frictionless. The angles θ_1 and θ_2 give the directions of these forces and they can be measured directly with a protractor. The tension $\vec{\mathbf{T}}_3$ is directed downwards and its magnitude is equal to the weight hanging at the end of the string (m_3g). So, using the usual directions for the x and y axes, equations 2 and 3 can be written as follows.

$$T_1 \cos\theta_1 = T_2 \cos\theta_2, \tag{4}$$

$$T_1 \sin\theta_1 + T_2 \sin\theta_2 = T_3. \tag{5}$$

To test the validity of these theoretical equations we need to compute the left and right sides of each equation (using experimental values) and find the percentage difference. A small percentage difference means good agreement between experiment and theory.

Some trials

To start with, you could use different sets of three weights for the three strings and measure the corresponding angles. Make a table showing the choices of the weights and the angles and the four quantities on the left and right sides of equations 4 and 5 in separate columns. Make a column each for the differences of left and right sides of the two equations. Finally, make columns for the two percentage differences. Note that the magnitudes of the tensions are computed by multiplying the values of the hanging masses by g, the acceleration due to gravity. As g is common for all terms in the equations, you may drop it from all equations. Also note that the masses of the hangers for the weights must be included.

Try and find conditions under which the percentage differences are the smallest and the largest. See if you can explain why.