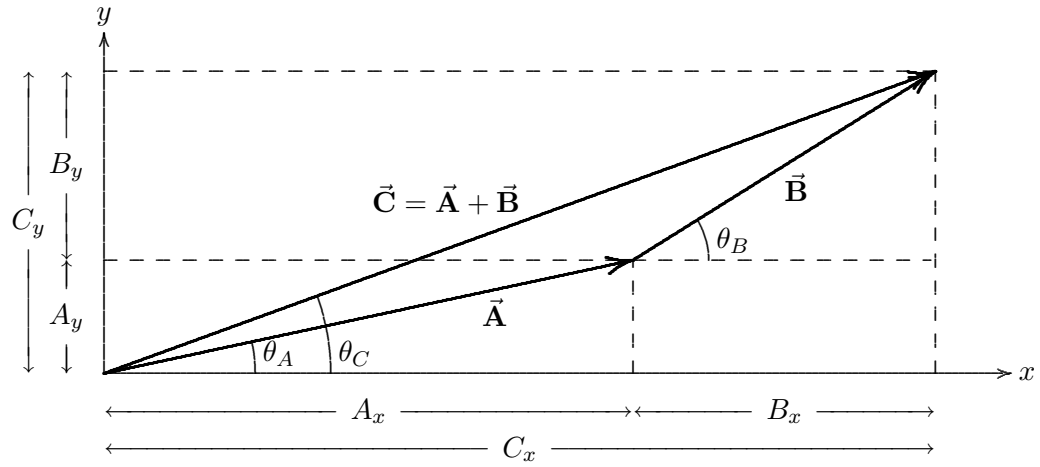
Vector Algebra*

Object

To compare graphical and numerical methods in vector algebra.

Theory

The sum of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ can be found graphically by shifting $\vec{\mathbf{B}}$ parallel to itself until its "tail" touches the "tip" of vector $\vec{\mathbf{A}}$. The sum of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ (call it $\vec{\mathbf{C}}$) is the vector starting at the "tail" of $\vec{\mathbf{A}}$ and ending at the "tip" of the shifted vector $\vec{\mathbf{B}}$ (see figure below).



The same result can be obtained by adding components of the vectors in some coordinate system. That is , if

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}, \tag{1}$$

then

$$C_x = A_x + B_x \tag{2}$$
$$C_y = A_y + B_y \tag{3}$$

where A_x and A_y are the x and y components of $\vec{\mathbf{A}}$ (and similarly for $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$). Hence,

$$A_x = A\cos\theta_A \quad , \quad A_y = A\sin\theta_A \tag{4}$$
$$B_x = B\cos\theta_B \quad , \quad B_y = B\sin\theta_B \tag{5}$$
$$C_x = C\cos\theta_C \quad , \quad C_y = C\sin\theta_C \tag{6}$$

where A, B and C are the magnitudes of the three vectors and θ_A, θ_B and θ_C are the respective angles with respect to the x-axis.

The measurement method

On a sheet of paper draw the coordinate axes x and y. Draw the two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ as shown in the figure above. Find the sum $\vec{\mathbf{C}}$ by joining the tail of $\vec{\mathbf{A}}$ to the tip of $\vec{\mathbf{B}}$. Measure the components of $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$. Using the measured values, compare the left and right sides of each of the two equations 2 and 3.

To find the components, you will have to draw the dashed lines as shown in the figure. Use a square or a protractor to get the right angles as accurately as possible.

You can also find the components using the equations 4, 5 and 6 if you measure the magnitudes (lengths) of the vectors and the angles they make with respect to the x axis.

Some trials

Check the component addition equations (equations 2 and 3) for the sum of two vectors. Find the components by direct measurement or by measuring the magnitudes and angles of the vectors and then using equations 4, 5 and 6.

Do the same for the difference of two vectors. Note that subtraction is the same as addition with the subtracted vector reversed in direction.

Extend the method for three or more vectors.