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RECONFIGURABLE K-ARY TREE MULTIPROCESSORS

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Abstract

In this paper, we present a strongly fault-tolerant design for the l-level k-ary tree multiprocessor and examine its reconfigurability. Our design assigns one spare node to the regular nodes of each subtree with l_c levels. Moreover, spare nodes are interconnected to form a spare tree. Our approach utilizes the circuit-switched capabilities of the communication modules of the spare nodes to tolerate a large number of faulty nodes and faulty links. Both theoretical and simulation results are presented. Compared with other proposed schemes, our approach can tolerate significantly more faulty nodes and faulty links with a low overhead and no performance degradation.

Key Words

Fault tolerance, $k-$ary tree, spare allocation, reconfiguration, augmented multiprocessor, circuit-switched

1. Introduction

The tree is a useful topology for networks of hierarchical computing systems and has been used in the design of multicomputers [1]. One attractive feature of the tree-based architecture with n-nodes is that any two nodes can exchange information in $O(\log n)$ time. A major disadvantage of this topology is that a single faulty node or faulty link can disable the operation of the entire tree. Hence, some form of fault tolerance is essential.

Fault-tolerant trees have been examined by a number of researchers. To sustain the same level of performance, some researchers have investigated hardware schemes where the spare nodes and spare links are used to replace faulty ones. In the literature, a reconfigurable system that retains the same service level, as well as keeping the same system topology after the occurrence of faults, is called a strongly fault-tolerant [2] system. A strongly fault-tolerant system can support the original task partition, and hence it can use faster deterministic routers instead of slower adaptive ones.

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An optimal 1-fault-tolerant k-ary tree with l-levels was proposed by Hayes [3]. The scheme for $k = 4$ and $l = 3$ is depicted in Fig. 1. Upon a node failure, using the spare nodes and the spare links, a k-ary tree with l-levels is maintained. Hence, the scheme is a strongly fault-tolerant one. Kwan and Toida [4] extended the approach to design an optimal 2-fault-tolerant binary tree. Other researchers [5–7] have proposed schemes to tolerate more faulty nodes, based on a covering technique where a node u is said to cover a node v if node u replaces node v when node v becomes faulty. Then the node covering u replaces u, and so on. The scheme in [5] requires a node degree of $m(k+1)$ to tolerate m faulty nodes in a k-ary tree. A 200% spare link overhead is required in [6] to tolerate one faulty node per level of the tree. The approach in [7] attains a slightly better result than that of [6] with 100% spare link overhead. To reduce the number of required spare links, some researchers [8–11] have proposed local reconfiguration schemes where a spare node is assigned to each set of nodes. One faulty node from each set is tolerated. The schemes have spare link requirements similar to ours.

Figure 1. Optimal 1-fault-tolerant tree proposed by Hayes.

In this paper we propose a global reconfiguration scheme that utilizes circuit-switched communication to make the k-ary tree strongly fault tolerant. In our scheme, one spare node is assigned to each group of regular nodes, called a cluster (Fig. 2); each spare node is connected to every regular node of its cluster via an intracluster spare link. Furthermore, the spare nodes of neighbouring clusters are interconnected using intercluster spare links; two clusters are declared neighbours if there exists at least one regular node in each with a direct link between them. We call the resulting topology the *enhanced cluster k-ary Tree* (ECKT). Our approach differs from others in the way we interconnect the spare nodes to the faulty regular nodes: we utilize the circuit-switched capabilities of various spare nodes' communication modules to construct edge-disjoint paths between multiple faulty regular nodes of a cluster and multiple unassigned spare nodes in different clusters. We use this property to show that our approach can tolerate multiple faulty nodes within a cluster. Hence, it has a higher fault-tolerant capability than other approaches. A similar technique is used to tolerate faulty links by utilizing

the spare links to establish parallel paths with them and bypassing them.

The rest of the paper is organized as follows. In the next section, notation and definitions are given. An overview of our approach is presented in Section 3. In Section 4 we examine the reconfigurability of the ECKT; both theoretical and simulation results are presented. Concluding remarks are given in Section 5.

2. Notation and Definitions

Each node of an l-level k-ary tree is represented by $P(i, j)$ where i is the level number and j is the index of the node in the level i. Similarly, the spare node j in the level i is denoted by $S(i, j)$. The link connecting any two nodes P and Q is represented by $P \rightarrow Q$. Finally, we define the *connection requirement* (C_R) of a spare node α in a cluster with multiple faulty nodes as the number of edge-disjoint paths that must be constructed, within the spare tree from spare node α to other spare nodes in the fault-free clusters, so that faulty nodes can be tolerated.

3. Overview of the ECKT

An ECKT with l levels is constructed by assigning one spare node to the regular nodes of each subtree with l_c levels. We call such a subtree a cluster. Then, the number of regular nodes in one cluster is $\frac{k^{l_c}-1}{k-1}$. The degree of a regular node at the root is $k+1$, at each internal node is $k+2$, and at each leaf node is 2. The network interconnecting the spare nodes (the spare tree) is connected as a k_s-ary tree with l_s levels, where $k_s = k^{l_c}$ and $l_s = \frac{l}{l_c}$. Hence, the number of spare nodes is $\frac{{k_s}^{l_s}-1}{k_s-1}$ and the number of spare links within the spare tree is $\frac{{k_s}^{(l_s-1)}-1}{k_s-1}k_s$. Figs. 2(a) and 2(b) depict 6-level enhanced cluster 2-ary trees with $l_c = 3$ and $l_c = 2$, respectively. In the figure, the regular links and the spare links are shown with heavy and light lines, respectively, and a cluster is highlighted with a dotted line. Note that the spare nodes in Fig. 2(a) are interconnected as an 8-ary tree ($k_s = 2^3$) with two levels ($l_s = \frac{6}{3}$), and in Fig. 2(b) they are interconnected as a 4-ary tree with three levels.

Figure 2. A 6-level enhanced cluster 2-ary tree with (a) $l_c = 2$ and (b) $l_c = 3$.

We next describe how an ECKT tolerates faulty nodes and faulty links. We assume that faulty nodes retain their ability to communicate. This is a common assumption as, in today's distributed memory multiprocessors, the computation and the communication modules of a node are separate. Furthermore, because the complexity of the computation module is much greater than that of the communication module, the probability of a failure in the computation module is much higher. This assumption may be avoided by duplicating the communication module in each node.

To facilitate reconfiguration, the routing channels described below are used at each node. The block diagrams of a regular node router and a spare node router for an ECKT are depicted in Fig. 3. Each node consists of a computation module and a communication module. The communication module is made of several routing channels, and each routing channel consists of a *channel in* and a *channel out*. For example, the routing channel 0 of the regular node depicted in Fig. 3(a) is made of Ch0 In and Ch0 Out. Each regular node router consists of $r+1$ routing channels, allowing it to connect to one of its r neighbouring regular nodes as well as its designated spare node. The connection to a neighbouring regular node is provided via ChX In and ChX Out channels, where $X = 0$ for the parent node, $X = 1$ for the leftmost child node, $X = 2$ for the next child node to the right, and so on. The connection to the designated spare node is facilitated using Spare In and Spare Out channels. Fig. 3(b) illustrates the communication module of a spare node with n regular nodes in its designated cluster and j neighbouring spare nodes. The node-routing channel PX (Node PX In and Node PX Out) allows the spare node to be connected to the regular node X within its cluster. Similarly, connection to another neighbouring spare node is provided using the appropriate Spare ChX In and Spare ChX Out.

Figure 3. Block diagram of (a) regular node, and (b) spare node.

Connecting a spare node to a regular node is done to tolerate a node failure. If the spare node resides in the cluster of the faulty node, the spare routing channel (Spare In and Spare Out) of the faulty node is connected to the appropriate node routing channel of the spare node. For example, in Fig. 2(a), if the node P(1, 1) becomes faulty the spare node S(0, 0) replaces it, as illustrated in Fig. 4. The heavy lines in the figure represent the permanent connections made after the reconfiguration. Hence, the computation module of the spare node replaces the computation module of the faulty node. In addition, the new communication module consists of the functional communication module of the faulty node merged with the appropriate routine channel of the spare node. Due to the circuit-switched capabilities of the communication modules, the cost of communication is nearly constant between any two given nodes. Therefore, replacing P(1, 1) with S(0, 0) has negligible effect in the performance of the system.

Figure 4. Spare node S(0,0) replacing faulty node P(1,1).

If the assigned spare node and the faulty node belong to different clusters, a dedicated path to connect them needs to be established. Such a path can be constructed by linking the appropriate routing channels of the intermediate spare nodes. For example, in Fig. 2(a), if the node P(2, 1) is faulty and the spare S(0, 0) is not available, the spare S(1, 1) can replace it by linking the appropriate spare routing channels of S(0, 0) and S(1, 1). Moreover, the proper node-routing channel of S(0, 0) is linked with the spare routing channel of P(2, 1). The dedicated path then becomes an extension of the communication module of the faulty node, and the spare node functionally replaces

the faulty one. Furthermore, due to the capabilities of the circuit-switched routing modules, the physical location of the faulty node and its assigned spare node becomes irrelevant.

There are three cases that may involve routing data through a faulty node. The first is when the message originates from the faulty node. Here, its spare replacement sends its data via its injection channel (Fig. 3(b)) out to the appropriate Node PX Out channel. The message is then routed to the communication module (CM) of the faulty node via Spare In. Depending on the final destination node, any of the channel outs may be selected. The second case is when the destination of the message is the faulty node. Once the data reach the CM of the faulty node, they are automatically routed to the channel Spare Out instead of the Consumption Channel. The spare node's CM would consequently receive the message using the appropriate channel (Node PX In). The message is then sent to the computation module of the spare node via the Consumption Channel. The third case is when the faulty node is neither the source nor the destination of the message, but is used merely as an intermediate switch connection. In this case, the spare node is not involved at all, and routing is done normally. Therefore, each spare node has dual functions: one is to be the logical replacement for a faulty node, and the other is to be an intermediate switch connection. Both functions may be active at the same time. Note that the connection of the spare node router to the active node(s) is established during the reconfiguration and remains intact thereafter.

Upon detecting a node failure, the spare node within the respective cluster logically replaces the faulty node. If the local spare node is not available, an available spare node from a different cluster may be used. Therefore, multiple logical spare nodes can exist in a cluster. Every spare node consequently activates its links to the active channels and disables the rest. The spare node's CM then forwards messages bound for other spare nodes as well as forwarding/receiving its data to/from other regular nodes via the CM of the faulty node as discussed above. Therefore, no modification to the available computation or communication algorithm is needed.

The ECKT can tolerate both intracluster and intercluster link failures. This is accomplished by utilizing the spare links to establish a parallel path to the faulty link. To tolerate an intracluster link failure, using the spare links and through the local spare node, a parallel path to the faulty link is established. For example, in Fig. 2(a), if the link $\mathrm{P}(3,1) \rightarrow \mathrm{P}(4,2)$ is faulty, it can be bypassed by the path $\mathrm{P}(3,1) \rightarrow \mathrm{S}(1,1) \rightarrow \mathrm{P}(4,2)$ as specified below. The CM of $\mathrm{S}(1,1)$ is configured so that the node-routing channel 0 and the node-routing channel 1 are interconnected. In addition, regular nodes $\mathrm{P}(3,1)$ and $\mathrm{P}(4,2)$ logically replace their routing channels, which connect them to the faulty link, with their spare routing channels. Consequently, all messages that are bound for the faulty link are sent through the path $\mathrm{P}(3,1) \rightarrow \mathrm{S}(1,1) \rightarrow \mathrm{P}(4,2)$. To tolerate an intercluster link failure, more than one spare node CM is utilized. For example, in Fig. 2(a), the path $\mathrm{P}(2,0) \rightarrow \mathrm{S}(0,0) \rightarrow \mathrm{S}(1,0) \rightarrow \mathrm{P}(3,0)$ can bypass the intercluster faulty link $\mathrm{P}(2,0) \rightarrow \mathrm{P}(3,0)$ as specified below. The node $\mathrm{P}(3,0)$ is connected to the spare node $\mathrm{S}(1,0)$ by linking $\mathrm{P}(3,0)$'s Spare In and Spare Out to $\mathrm{S}(1,0)$'s Node P0 Out and Node P0 In, respectively. Similarly, the spare routing channel of the node $\mathrm{P}(2,0)$ is connected to the node P1 routing channel of the spare node $\mathrm{S}(1,0)$. Finally, the spare nodes $\mathrm{S}(0,0)$ and $\mathrm{S}(1,0)$ are interconnected by linking their appropriate spare channel outs and spare channel ins. Reconfiguration fails if either of the two nodes at the end of a faulty link is also faulty, because the pertinent spare link has to be used to tolerate both the faulty node and the faulty link. The reconfiguration also fails if two faulty links have a common regular node, as the spare link that connects to the shared regular node has to be used in more than one dedicated parallel path.

Fig. 5 illustrates the reconfiguration of the ECKT in Fig. 2(a) in the presence of indicated faulty nodes and faulty links. For the sake of clarity, in Fig. 5 nonactive spare links are deleted and active spare links are drawn in a variety of line styles to distinguish the bypass paths. As shown in the figure, spare nodes $\mathrm{S}(0,0)$, $\mathrm{S}(1,1)$, $\mathrm{S}(1,6)$, $\mathrm{S}(1,7)$, $\mathrm{S}(1,3)$, $\mathrm{S}(1,0)$, $\mathrm{S}(1,2)$, $\mathrm{S}(1,4)$, and $\mathrm{S}(1,5)$ logically replace $\mathrm{P}(1,1)$, $\mathrm{P}(2,1)$, $\mathrm{P}(2,3)$, $\mathrm{P}(3,7)$, $\mathrm{P}(4,8)$, $\mathrm{P}(5,2)$, $\mathrm{P}(5,9)$, $\mathrm{P}(5,19)$, and $\mathrm{P}(5,20)$, respectively. Also, intracluster faulty links $\mathrm{P}(3,1) \rightarrow \mathrm{P}(4,2)$, $\mathrm{P}(4,4) \rightarrow \mathrm{P}(5,8)$, $\mathrm{P}(3,3) \rightarrow \mathrm{P}(4,7)$, and intercluster faulty link $\mathrm{P}(2,0) \rightarrow \mathrm{P}(3,0)$ are bypassed using parallel paths $\mathrm{P}(3,1) \rightarrow \mathrm{S}(1,1) \rightarrow \mathrm{P}(4,2)$, $\mathrm{P}(4,4) \rightarrow \mathrm{S}(1,2) \rightarrow \mathrm{P}(5,8)$, $\mathrm{P}(3,3) \rightarrow \mathrm{S}(1,3) \rightarrow \mathrm{P}(4,7)$, and $\mathrm{P}(2,0) \rightarrow \mathrm{S}(0,0) \rightarrow \mathrm{S}(1,0) \rightarrow \mathrm{P}(3,0)$, respectively. Considering only the faulty nodes in Fig. 5 and examining the cluster associated with the spare $\mathrm{S}(0,0)$, we see that two out of three faulty nodes of the cluster have to be assigned to available spare nodes from other fault-free clusters via edge-disjoint paths through the spare $\mathrm{S}(0,0)$. Therefore, the C_R (connection requirement) of the spare $\mathrm{S}(0,0)$ is 2. Note that the C_R of a spare node is equal to the number of faulty nodes that reside within its cluster, minus one.

Figure 5. The ECKT in presence of faults.

4. Reconfigurability of the ECKT

4.1 Reconfiguration under Faulty Nodes

Let us group the spare nodes into three sets: S_S (set of source nodes), S_U (set of used nodes), and S_T (set of target nodes). A source node is a spare node in a cluster with multiple faulty nodes. The set S_S then represents the spare nodes with a C_R greater than 0. S_T is the set of unassigned spare nodes, and S_U consists of spare nodes that have been assigned to faulty nodes and have a C_R of 0. For example, considering only the faulty nodes in Fig. 5, after assigning the local spare node to a local faulty node in each faulty cluster, $\mathrm{S}_S = \{\mathrm{S}(0,0)\}$, $\mathrm{S}_U = \{\mathrm{S}(1,0), \mathrm{S}(1,2), \mathrm{S}(1,4), \mathrm{S}(1,5), \mathrm{S}(1,7)\}$, and $\mathrm{S}_T = \{\mathrm{S}(1,1), \mathrm{S}(1,3), \mathrm{S}(1,6)\}$. During the reconfiguration algorithm, which is discussed later in this section, the spare nodes are dynamically assigned to the various sets. To illustrate this, suppose the C_R of a spare node $\alpha \in \mathrm{S}_S$ is greater than 0 and there is a

dedicated path from α to $\beta \in S_T$. Consequently, β replaces a faulty node in the cluster of α via the dedicated path. β is then called used and is assigned to S_U. Also, the C_R of α is reduced by one. If the C_R of α becomes zero, it is also marked as used and is assigned to S_U. The ECKT is called reconfigured when S_S becomes an empty set.

From the previous section, it follows that the reconfigurability of the ECKT is a function of the number of dedicated and edge-disjoint paths, within the spare tree, that can be established between the local spare nodes (nodes in S_S) of the clusters with multiple faulty nodes and the available spare nodes (nodes in S_T) of the fault-free clusters. However, spare nodes do not have to be interconnected as a tree. Obviously, if the spare nodes are interconnected as a complete graph, the ECKT can tolerate up to $\frac{k_s{}^{l_s}-1}{k_s-1}$ faulty nodes regardless of their distribution. Hence, the reconfigurability of the ECKT is a direct consequence of the connectivity of the topology that interconnects the spare nodes. Let us represent the topology of the graph connecting the spare nodes by $G = (V, E)$ where $V = S_S \bigcup S_U \bigcup S_T$ and E consists of the appropriate spare links. Let the C_R of a node $n \in S_S$ be represented by $C_R(n)$, and denote the sum of the C_R's of all nodes in a set P as $\sum_{n\in P} C_R(n)$. As the number of faulty nodes cannot exceed the number of spare nodes, $|S_T| \geq \sum_{n\in S_S} C_R(n)$. The following theorem examines the connectivity of G as it pertains to the reconfigurability of the ECKT.

Theorem 1. Consider a graph $G(V, E)$ where $V = S_S \bigcup S_U \bigcup S_T$. The necessary and sufficient condition for every node $n \in S_S$ to have $C_R(n)$ edge-disjoint paths to $C_R(n)$ nodes in S_T is that the minimum number of edges leaving any subset of nodes $P \subseteq V$ be greater than or equal to $\sum_{n\in(P \bigcap S_S)} C_R(n) - |P \bigcap S_T|$.

Proof. We first prove the necessary condition: if from every node $n \in S_S$, there exists $C_R(n)$ edge-disjoint paths to $C_R(n)$ nodes in S_T, then the minimum number of edges leaving any subset of nodes $P \subseteq V$ must be greater than or equal to $\sum_{n\in(P \bigcap S_S)} C_R(n) - |P \bigcap S_T|$, which is the sum of the C_R's of S_S nodes within P minus the number of S_T nodes in P.

Let us consider a subset $P_1 \subseteq S_S$. Each of the edge-disjoint paths from a node in S_S to a node in S_T must be carried over at least one edge in the cutset $(P_1, V - P_1)$. Therefore, the sum of the C_R's of the nodes in P_1, which represents the total required number of edge-disjoint paths from the nodes in P_1 to the nodes in S_T, must be smaller than or equal to the number of edges in the cutset $(P_1, V - P_1)$. Now, let us consider a subset $P \subseteq V$ and denote the graph interconnecting the nodes of P as g. Obviously, g is a subgraph of G. Within g, there exists only $|P \bigcap S_T|$ target nodes. Therefore, at most $|P \bigcap S_T|$ of the edge-disjoint paths may exist in g. The rest of the paths must then be carried over the cutset $(P, V - P)$. Therefore, the necessary condition follows.

We next prove the sufficient condition: if the minimum number of edges leaving any subset of nodes $P \subseteq V$ is greater than or equal to $\sum_{n\in(P \bigcap S_S)} C_R(n) - |P \bigcap S_T|$, every node $n \in S_S$ would have $C_R(n)$ edge-disjoint paths to $C_R(n)$ nodes in S_T. Let us create a new graph $G' = (V', E')$ by adding two nodes s and t to G as specified below and depicted by Fig. 6. Each node in S_T is connected to t via a single edge. Each node $n \in S_S$ is connected to s via $C_R(n)$ parallel edges. Let the sum of the C_R of all nodes in S_S be L. Number of edge-disjoint paths between s and t in G′, according to Menger's theorem [12], is equal to the size of the mincut in G'. We will show that there always exists an (s, t) mincut in G' whose size is equal to L. The mincut in G' may exist at s, t, G, or some combination of them. By construction, the size of the cut at s equals L. Similarly, the cutsize at t is greater than or equal to L because $|S_T| \geq \sum_{n\in S_S} C_R(n) = L$. Per stated condition, for $P = s \bigcup S_S$ or $P = s \bigcup S_S \bigcup S_U$, the cut $(P, V' - P)$ must have a cutsize greater than or equal to L. Consider a general cut in G' crossing L_1 of the edges connecting s to S_S nodes, L_2 edges of G, and L_3 of the edges connecting S_T nodes to t (Fig. 6). The number of S_T nodes on the unshaded side of the cut is L_3. The sum of the C_R's of S_S nodes within the same side of the cut is $L - L_1$. Therefore, the stated condition can be formulated as $L_2 \geq (L - L_1) - L_3$ or $L_1 + L_2 + L_3 \geq L$. From this inequality, it follows that any cut in G' has a cutsize greater than or equal to L. Therefore, L is the size of the mincut. Hence, there exist L edge-disjoint paths between nodes s and t. Each of these s-t edge-disjoint paths must pass through a unique node in S_T because each node in S_T is connected to t via a igle edge. As there only exists L edges from s (one per path), the number of edge-disjoint paths from s that passes through each node $n \in S_S$ is equal to $C_R(n)$. Therefore, each node $n \in S_S$ can make $C_R(n)$ edge-disjoint paths to $C_R(n)$ distinct nodes in S_T. QED

Figure 6. A cut in graph G'.

We next apply Theorem 1 to the spare tree and examine the reconfigurability of the ECKT.

Theorem 2. The necessary condition for the ECKT to reconfigure in the presence of faulty nodes is that the number of faulty nodes in each subtree with l_i levels $(l_i = j \times l_c; j = 1, 2, \cdots)$, be less than or equal to $\frac{k_s^{\frac{l_i}{l_c}}-1}{k_s-1} + 1$.

Proof. Let us examine the ECKT by starting at the leaf nodes and moving towards the root node. The number of spare nodes of a subtree with $l_i = j \times l_c$ levels is $\frac{k_s^j-1}{k_s-1}$; two such subtrees with $l_i = l_c$ and $l_i = 2l_c$ are depicted in Fig. 7, with dotted lines around the smaller and larger subsets, respectively. Because only one additional intercluster spare link exists that connects the subtree to an external spare node, the maximum number of faulty nodes that can be tolerated in the subtree is $\frac{k_s^j-1}{k_s-1} + 1$. QED

Figure 7. An ECKT with $k = 2$, $l = 6$, and $l_c = 3$.

From examination of the cluster shown by the dotted line in Fig. 7, it is obvious that Theorem 1 is violated for a cluster with more than two faulty nodes; the C_R of the local spare node must be at most one, as only one intercluster spare link is crossed. Other examples of the ECKT will yield similar results. Therefore, under a fixed

number of faulty nodes per cluster, no theoretical lower bound on the number of tolerated faulty nodes per cluster can be established.

We next examine the simulation results based on the following reconfiguration algorithm. An optimal reconfiguration algorithm can be developed by utilizing the maxflow algorithm. Here, optimality is measured as the ability to assign a spare node to every faulty node whenever such an assignment is feasible vis-à-vis Theorem 1. The main drawback to a reconfiguration using the above algorithm is that a digraph representation of the spare network has to be constructed [11] and the spare node assignment has to be done by the host processor. To overcome these deficiencies, we next present a near-optimal reconfiguration algorithm, which is called *Alloc-Spare*. The algorithm is near optimal, as there can be cases where the reconfiguration fails even though Theorem 1 holds; these cases are extremely rare and we did not encounter any in our simulation runs. The algorithm consists of three parts:

1. *Early Abort:* A solvability test based on Theorem 2 is performed to determine whether the reconfiguration is feasible.
2. *Local Assignment:* The local spare node of every faulty cluster is assigned to a faulty node within the cluster. If all faulty nodes are covered, the ECKT is reconfigured.
3. *Nonlocal Assignment:* To find a set of candidate spare nodes that can be assigned to a faulty node, we utilize Lee's path-finding algorithm [13]. The algorithm begins by constructing a breadth-first search of minimum depth d $(1 \leq d \leq 2(l_s - 1))$ in the spare tree from the local spare node of a faulty cluster with a nonzero C_R. If a free spare node is found, a path is formed to the source node. The algorithm guarantees that a path to a spare node will be found if one exists and that the path will be the shortest possible [13]. Therefore, all faulty nodes that are one link away from available spare nodes (at depth 1) are assigned first. Once a path is formed, the links associated with that path are deleted from the spare tree, resulting in a new structure. If there still remain some uncovered faulty nodes, a solvability test to check the C_R of neighbouring spare nodes, similar to Early Abort, is performed on the new structure, and Step 3 is repeated for a higher depth d. Reconfiguration fails if $d > 2(l_s - 1)$, which is the longest acyclic path in the spare tree. QED

Alloc-Spare may be applied distributively. Step 1 can be performed by having each spare node check its own node degree as dictated by Theorem 2 and broadcast that information along with its C_R to its neighbouring spare nodes. Each spare node consequently checks for solvability based on Theorem 1. Step 2 is done by the local spare node of each faulty cluster. For Step 3, each spare node with a nonzero C_R makes a breadth-first search within the spare tree to locate the unassigned spare node(s).

We implemented algorithm *Alloc-Spare* for an enhanced cluster 3-ary tree with $l = 9$ and $l_c = 3$. Hence, 757 spare nodes are interconnected as a 27-ary spare tree with $l_s = 3$.

Figure 8. Tolerating faulty nodes only.

The simulation result for up to 400 randomly placed faulty nodes is shown in Fig. 8. One thousand simulation runs were performed for each given number of faulty nodes. The other plot in the figure pertains to the result of the local reconfiguration scheme [8–11]. The result indicates nearly 100% reconfigurability for the ECKT in the presence of about 30 faulty nodes and nearly 90% reconfigurability in the presence of nearly 100 faulty nodes. The approaches in [7] and [6] perform slightly worse and slightly better than the local reconfiguration scheme, respectively. Finally, the approach proposed in [4] tolerates only two faulty nodes, and the scheme in [5] tolerates a fixed number of faulty nodes at the expense of a large node degree.

Figure 9. Reconfigurability of ECT under different node degree and cluster size.

To examine the effect of node degree and cluster size on the fault tolerance of the ECKT, the reconfigurability of three ECKT's were compared. Their simulation results are shown in Fig. 9. The three plots in Fig. 9 belong to three ECKT's with G1:$\{k = 3,\ l = 9,\ l_c = 3,\ k_s = 27,\ l_s = 3\}$; G2: $\{k = 2,\ l = 12,\ l_c = 4,\ k_s = 16,\ l_s = 3\}$; and G3: $\{k = 2,\ l = 16,\ l_c = 4,\ k_s = 16,\ l_s = 4\}$. Our simulation results reveal that for a given ECKT, the best reconfiguration is achieved when the degree of the spare tree ($k_s = k^{l_c}$) and the spare tree levels ($l_s = \frac{l}{l_c}$) are kept to a minimum. However, the former requires l_c to be the minimum and the latter needs it to be the maximum. Hence, for a given ECKT, l_c should be selected such that there is a balance.

4.2 Reconfiguration under Faulty Links and Nodes

If two or more faulty links share a regular node in an ECKT, the reconfiguration fails because the spare link connecting the local spare node to the common node has to be shared by more than one dedicated path. For example, in Fig. 7, if links P$(2,0)$ →P$(3,0)$ and P$(2,0)$ →P$(3,1)$ are faulty, the spare link P$(2,0)$ →S$(0,0)$ has to be used by both of them, which is not possible. Therefore, no theoretical lower bounds on the number of tolerated faulty links can be established; only experimental results based on random distribution of faulty links can be examined.

Our reconfiguration algorithm, called the Minimum Parallel Path, utilizes unassigned spare links to establish a parallel path to each faulty link; if any one of the spare links of a parallel path to any faulty link is unavailable, the reconfiguration fails. We implemented the algorithm for an enhanced cluster 3-ary tree with $l = 9$ and $l_c = 3$. The simulation result for up to 200 randomly placed faulty links is shown in Fig. 10. The result indicates that the given ECKT can tolerate nearly 20 faulty links 90% of the time.

Figure 10. Tolerating link failures only.

Combining the reconfiguration algorithm, which toler-

ates faulty nodes and faulty links, allows a combination of faulty nodes and faulty links to be tolerated. We assumed the probability of a link failure to be the same as a node failure. After randomly allocating the faulty elements, the algorithm first tries to tolerate all faulty links. If successful, it then uses Alloc-Spare to tolerate the faulty nodes. The simulation result for an enhanced cluster 3-ary tree with $l = 9$ and $l_c = 3$ is shown in Fig. 11. The result indicates that nearly 50 faulty elements are tolerated by the ECKT 90% of the time.

Figure 11. Tolerating faulty nodes and links.

5. Conclusion

We have presented a strongly fault-tolerant design for the k-ary tree multiprocessor and examined its reconfigurability. Our scheme can tolerate both faulty nodes and faulty links without the need to alter the communication or computation algorithms. Although our scheme cannot guarantee a theoretical lower bound on the number of tolerated faulty nodes or faulty links, our simulation results, under random distribution of faulty nodes and faulty links, indicates that the ECKT could tolerate a relatively large number of faulty nodes and faulty links. Compared to other proposed schemes, the ECKT can tolerate significantly more faults for the same overhead.

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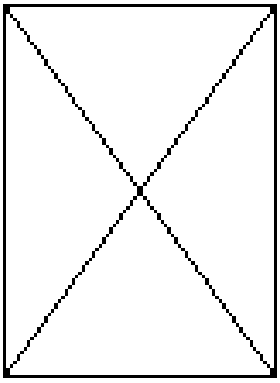
Biographies



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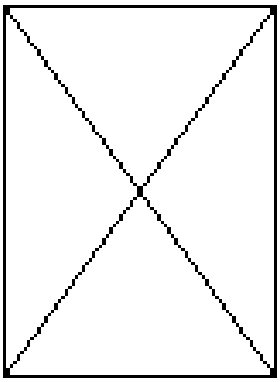

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