EGE -534 HW #2 Dr. Izadi

Name: ________________Key____

Problem 1 (10 Points)
Design a totally self-checking checker with 8 inputs.

$f = A \oplus B = \bar{A}B + A\bar{B}$

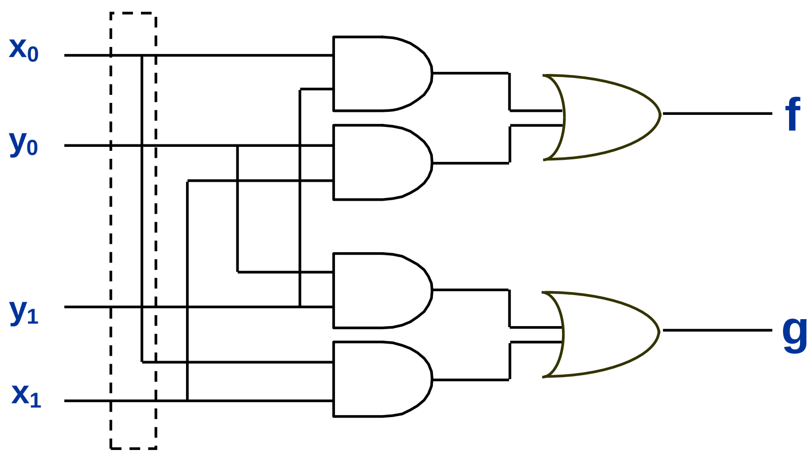
$g = \overline{A \oplus B} = \bar{A}\bar{B} + AB$

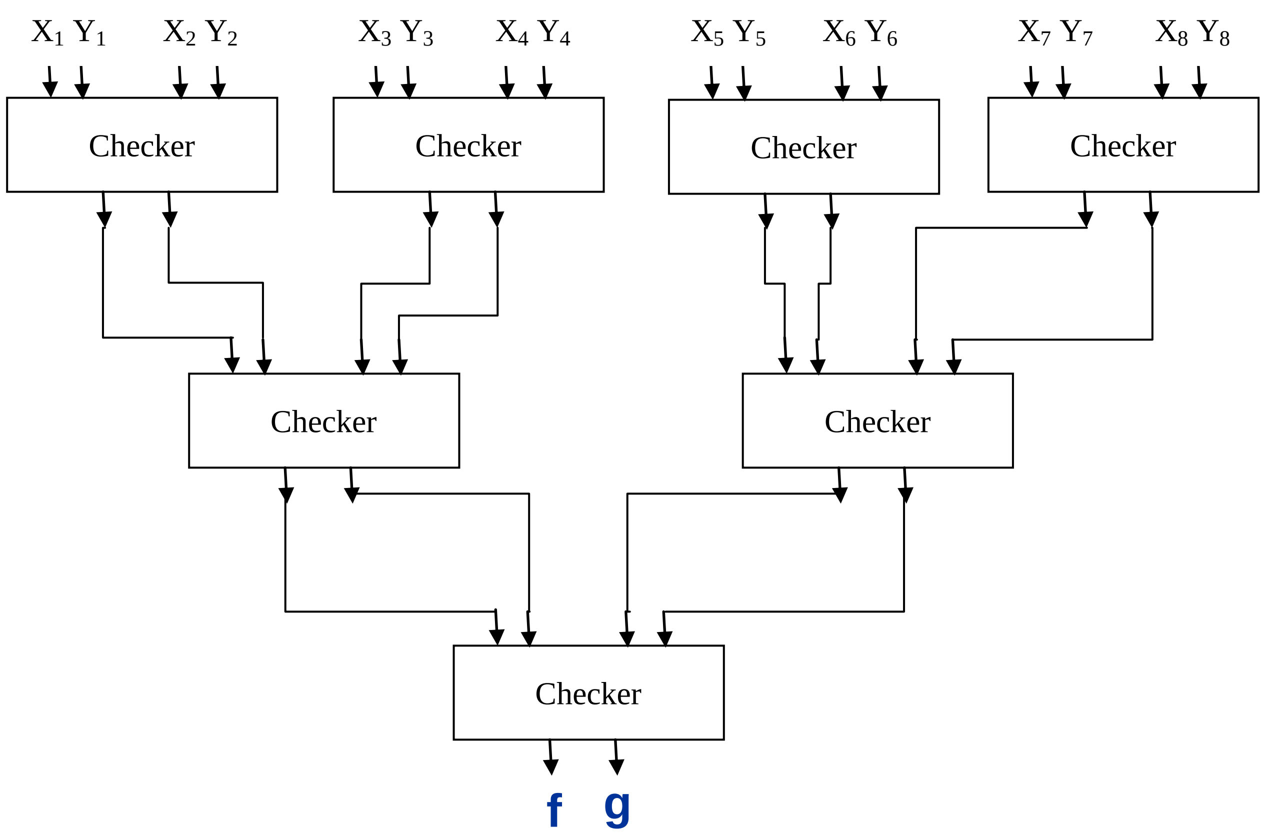
$A = \bar{x}_0 = y_0$

$B = \bar{x}_1 = y_1$

$f = x_0 y_1 + x_1 y_0$

$g = x_0 x_1 + y_1 y_0$





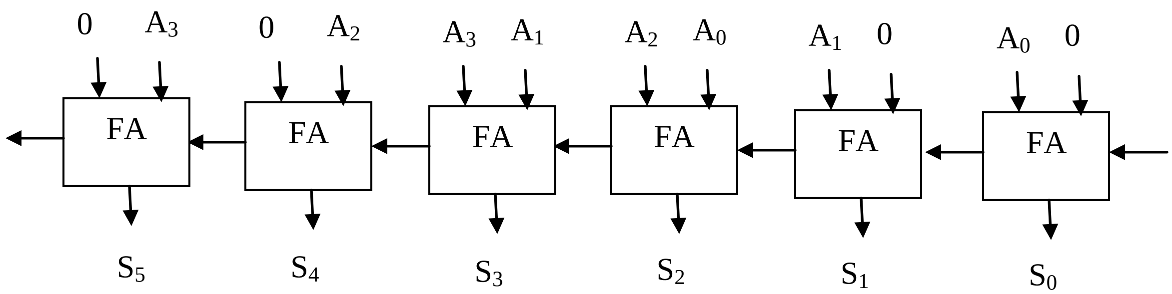
Problem 2 (20 Points)
Using full adders and basic gates, design a 5N code encoder, where N is a 4-bit binary number.

We can obtain 5N =N+4N

4N is obtained by appending a 00 to the LSBs of N.

	4N=	A_3	A_2	A_1	A_0	0	0
+	N=	0	0	A_3	A_2	A_1	A_0
	C_{out}	S_5	S_4	S_3	S_2	S_1	S_0

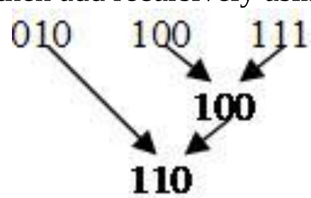
We can now add using 6 FA's



Problem 3 (20 Points)
Consider a low-cost residue code based on module 7.

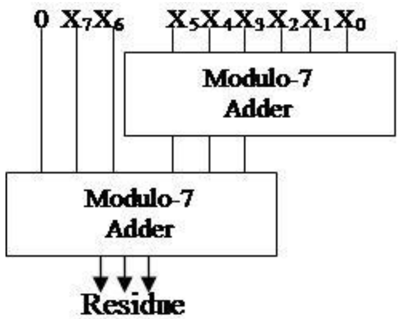
a. Show how do you obtain residue-7 check bits of $X_7\ X_6\ X_5\ X_4\ X_3\ X_2\ X_1\ X_0$ using recursive addition technique?
b. What is the theoretical base for this easy encoding process?

In A low cost residue code the modulo $m = 2^b-1$ if m=7 as in this case $7=2^3-1$ $\therefore$b=3. We group the bits into groups of b (3) bits, we then add recursively using modulo 7 as shown below:



$|167|_7 = 6 = 110$

A circuit to produce this correct residue is shown below:



B) What is the theoretical basis for this easy encoding process?

The basis is best shown via a table showing bit position weights grouped in groups of b bits. As we can see there is a repeating pattern when the bits are properly grouped.

256	128	64	32	16	8	4	2	1
X_8	X_7	X_6	X_5	X_4	X_3	X_2	X_1	X_0
8^2x4	8^2x2	8^2x1	8^1x4	8^1x2	8^1x1	8^0x4	8^0x2	8^0x1

$8^2(X_8\times4+X_7\times2+X_6\times1)$ $8^1(X_5\times4+X_4\times2+X_3\times1)$ $8^0(X_2\times4+X_1\times2+X_0\times1)$

(8^2) Mod 7 = (8^1) Mod 7 = (8^0) Mod 7 = 1

Therefore, the sum in Mod 7 will be

$$\begin{array}{ll} & X_2\times4+X_1\times2+X_0\times1 \\ & X_5\times4+X_4\times2+X_3\times1 \\ + & X_8\times4+X_7\times2+X_6\times1 \\ & \overline{\qquad\qquad\qquad\qquad}_7 \end{array}$$

Problem 4 (15 Points)
Convert 0 to 19 to RNS using modules [4,5,7]. Within this range demonstrate if the code is single error detecting. Repeat the same for single error correcting.

Value	RNS Value		
	m_1=4	m_2=5	m_3=7
0	000	000	000
1	001	001	001
2	010	010	010
3	011	011	011
4	000	100	100
5	001	000	101
6	010	001	110
7	011	010	000
8	000	011	001
9	001	100	010
10	010	000	011
11	011	001	100
12	000	010	101
13	001	011	110
14	010	100	000
15	011	000	001
16	000	001	010
17	001	010	011
18	010	011	100
19	011	100	101

Minimum number of bit differences between any two rows is two. Therefore, code distance is two. Hence, the code is capable of single error detecting.

Problem 5 (20 Points)
Consider a random-access memory that has a word format $X_4\,X_3\,X_2\,X_1\,X_0$ of size 5 bits. We can use Hamming code to correct any single bit in this memory.

a) What is the H (or P) matrix?
b) Given the four syndromes s_i computed by your SEC Hamming code for single-bit errors affecting data bit x_i, $0 \le i \le 4$. Also give the error-free syndrome s^*.
c) Explain how you would modify the SEC code you have defined above in order to obtain an SEC/DED code.

a) H matrix so that error code by SEC Hamming Code specifies bit position of the error.
$X_4X_3X_2X_1X_0$
$2^c \geq c+n+1$
$2^c \geq c+5+1$
$2^4 \geq 4+6$
$16 \geq 10$
$\therefore$ # of check bits =4
$\therefore$ Total length of codeword is 9 (4+5)
The syndrome can be used to point directly to the erroneous bit if the bits are arranged properly

9	8	7	6	5	4	3	2	1
X_4	P_3	X_3	X_2	X_1	P_2	X_0	P_1	P_0

As shown above the value of the syndrome now locates specifically the bit that is in error

	9	8	7	6	5	4	3	2	1
	X_4	**P_3**	**X_3**	**X_2**	**X_1**	**P_2**	**X_0**	**P_1**	**P_0**
C_3	1	1	0	0	0	0	0	0	0
C_2	0	0	1	1	1	1	0	0	0
C_1	0	0	1	1	0	0	1	1	0
C_0	1	0	1	0	1	0	1	0	1

Error free syndrome is $C_3C_2C_1C_0$=0000 b.

Error Bit	Syndrome
P_0	0001
P_1	0010
P_2	0100
P_3	1000
X_0	0011
X_1	0101
X_2	0110
X_3	0111
X_4	1001
No Error	0000

C) The above set up erroneously corrects faulty bits using basic hamming code. To overcome the problem by providing a code that can correct single bit errors and detect double bit errors, the modified hamming code is used. The modification consists of adding 1 parity check bit that checks parity of the entire hamming code word as shown.

X_4	X_3	X_2	X_1	X_0	C_3	C_2	C_1	C_0	C_4
				Data Bits	Check Bits				Parity bit

No Error	Check Bits 0 Check Bit 4 Good
Single Bit Error	Check Bits bad Check Bit 4 Bad
Double Bit Error	Check Bits bad Check Bit 4 Good

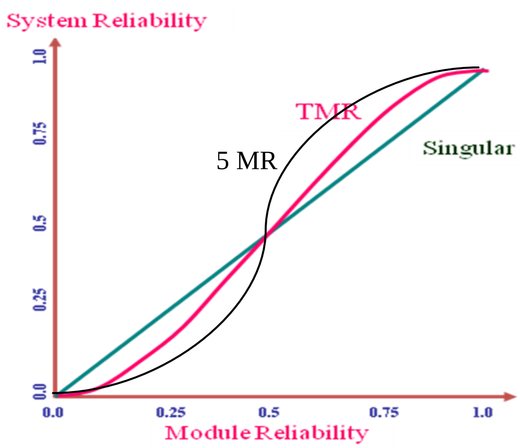
$X_4 \ldots X_1, X_0$	$p_3 \ldots p_0$	p_4
Single bit error	Detects error	Detects error
Double bits error	Detects error	Does not detect error

Problem 6 (20 Points)
Using the combinatorial model, determine the reliability of a simplex, TMR, and 5MR systems as a function of reliability of a simplex system, R(t) . You may assume a fault-free voter. Using MathLab, plot the reliability of the three systems versus R and comment on their relative reliabilities.

$$Rsimplex = R(t)$$

$$R_{TMR}(t) = 3R^2(t) - 2R^3(t)$$

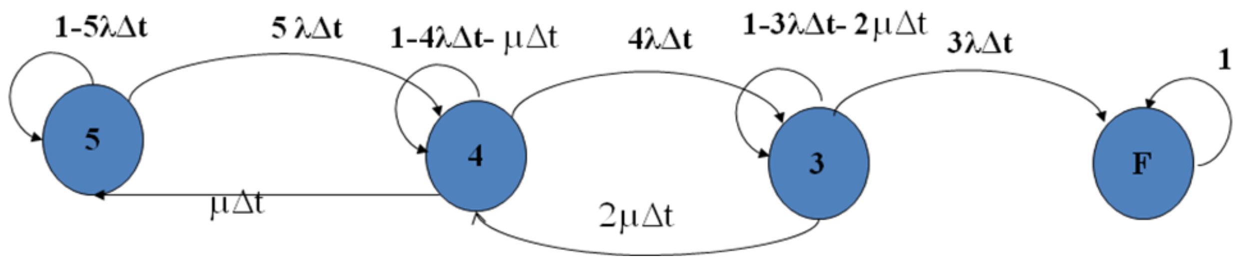
$$R_{5MR}(t) = 6R^5(t) - 15R^4(t) + 10R^3(t)$$



Problem 7 (25 Points)

Using Markov model, determine the discrete solution for the reliability of a 5MR system with λ failure rate and μ repair rate. You may assume that the system initially is fault free. Moreover, you may assume that once the 5MR has 3 more faulty modules, it enters a failed state that can't be repaired. Using MathLab plot R(t) from 0 to 5 hours using

a. $\Delta t = 0.01$, $\lambda = .001$ and $\mu. = .1$
b. $\Delta t = 0.01$, $\lambda = .01$ and $\mu. = .1$
c. $\Delta t = 0.01$, $\lambda = .001$ and $\mu. = .01$



$$\begin{bmatrix} P_5(t+\Delta t) \\ P_4(t+\Delta t) \\ P_3(t+\Delta t) \\ P_F(t+\Delta t) \end{bmatrix} = \begin{bmatrix} (1-5\lambda\Delta t) & \mu\Delta t & 0 & 0 \\ 5\lambda\Delta t & (1-4\lambda\Delta t-\mu\Delta t) & 2\mu\Delta t & 0 \\ 0 & 4\lambda\Delta t & (1-3\lambda\Delta t-2\mu\Delta t) & 0 \\ 0 & 0 & 3\lambda\Delta t & 1 \end{bmatrix} \begin{bmatrix} P_5(t) \\ P_4(t) \\ P_3(t) \\ P_F(t) \end{bmatrix}$$

- In compact form $\mathrm{P}(t+\Delta t) = \mathrm{A}\ \mathrm{P}(t)$
- At t = 0 $\mathrm{P}(\Delta t) = \mathrm{A}\ \mathrm{P}(0)$

$$\mathrm{P}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

At t =2 Δt

$$\mathrm{P}(2\Delta t) = \mathrm{A}\ \mathrm{P}(\Delta t) = \mathrm{A}^2\ \mathrm{P}(0)$$

- At t =n Δt $\mathrm{P}(n\Delta t) = \mathrm{A}^n\ \mathrm{P}(0)$

$R_{5MR}(t) = P_5(t) + P_4(t) + P_3(t)$