Solutions

Chapter 1

Problem 2

Choose the x coordinate in the initial directions of motion of the two particles. Choose the y direction perpendicular to the x direction. Then the momentum conservation equations give,

$$\begin{aligned} mv + (m/2)(v/2) &= mv_1' + (m/2)v_2' \cos\phi, & (1) \\ 0 &= (m/2)v_2' \sin\phi, & (2) \end{aligned}$$

where ϕ is the angle of deflection of the second particle. The second of the above equations shows that $\phi = 0$. Hence the first equation gives,

$$5v/4 = v_1' + v_2'/2. \quad (3)$$

The energy conservation equation gives,

$$(1/2)mv^2 + (1/2)(m/2)(v/2)^2 = (1/2)mv_1'^2 + (1/2)(m/2)v_2'^2. \quad (4)$$

Hence,

$$9v^2/8 = v_1'^2 + v_2'^2/2. \quad (5)$$

Solving for v_1' from equation 3 and squaring it gives.

$$v_1'^2 = 25v^2/16 + v_2'^2/4 - 5vv_2'/4. \quad (6)$$

Replacing this in equation 5 gives,

$$9v^2/8 = 25v^2/16 + v_2'^2/4 - 5vv_2'/4 + v_2'^2/2. \quad (7)$$

This produces the following quadratic equation.

$$12v_2'^2 - 20vv_2' + 7v^2 = 0. \quad (8)$$

The two solutions for this equation are $v_2' = 7v/6$ and $v_2' = v/2$. The second solution describes the case of no collision as the final velocity of the second particle is the same as its initial velocity. Hence, for a real collision, using

$$v_2' = 7v/6 \quad (9)$$

in equation 3 gives,

$$v_1' = 2v/3. \quad (10)$$

Problem 3

Choose x coordinate along the initial momentum of the first particle. Choose the $x-y$ plane such that the initial momentum and the final momentum are both in it. Momentum conservation equations in the $x-y$ plane gives

$$\begin{aligned} mv &= mv_1' \cos 30^\circ + mv_2' \cos\phi && (11) \\ 0 &= mv_1' \sin 30^\circ + mv_2' \sin\phi && (12) \end{aligned}$$

where ϕ is the angle of deflection of the second particle after collision. Energy conservation gives

$$mv^2/2 = mv_1'^2/2 + mv_2'^2/2 \tag{13}$$

As m is common to all terms, the three equations above become:

$$\begin{aligned} v &= v_1' \cos 30^\circ + v_2' \cos\phi && (14) \\ 0 &= v_1' \sin 30^\circ + v_2' \sin\phi && (15) \\ v^2 &= v_1'^2 + v_2'^2 && (16) \end{aligned}$$

These are three equations in the three unknowns v_1', v_2' and ϕ. First we eliminate ϕ from the first two equations by writing them as follows

$$\begin{aligned} v_2' \cos\phi &= v - v_1' \cos 30^\circ && (17) \\ v_2' \sin\phi &= -v_1' \sin 30^\circ && (18) \\ & && (19) \end{aligned}$$

and then squaring both sides of each equation and adding the two. This gives

$$v_2'^2 = v^2 + v_1'^2 - 2vv_1' \cos 30^\circ \tag{20}$$

Using equation 16 to eliminate $v_2'^2$ from the above equation gives

$$v_1' = v \cos 30^\circ \tag{21}$$

Replacing this in equation 16 gives

$$v_2' = v \sin 30^\circ \tag{22}$$

Replacing the above values of v_1' and v_2' in equations 14 and 15 gives

$$\cos\phi = \sin 30^\circ, \quad \sin\phi = -\cos 30^\circ \tag{23}$$

Hence,

$$\phi = -60^\circ. \tag{24}$$

Problem 4

Let the final velocities of the two particles be written as follows.

$$\vec{\mathbf{v}}'_1 = v'_1(\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}), \quad \vec{\mathbf{v}}'_2 = v'_2(\cos\phi\hat{\mathbf{i}} + \sin\phi\hat{\mathbf{j}}), \tag{25}$$

where v'_1 and v'_2 are their respective magnitudes. θ is the final angle of deflection of the first particle and $\phi = 60^\circ$ is the angle of deflection of the second particle.

As the masses are equal, the momentum conservation equations give,

$$2v/\sqrt{2} = v'_1\cos\theta + v'_2\cos\phi, \quad 0 = v'_1\sin\theta + v'_2\sin\phi. \tag{26}$$

The magnitudes of the initial velocities are,

$$v_1 = \sqrt{(v/\sqrt{2})^2 + (v/\sqrt{2})^2} = v, \quad v_2 = \sqrt{(v/\sqrt{2})^2 + (v/\sqrt{2})^2} = v \tag{27}$$

Hence, the energy conservation equation gives,

$$2v^2 = v'^2_1 + v'^2_2. \tag{28}$$

This gives three equations for the three unknowns v'_1, v'_2 and θ. To eliminate θ from the momentum conservation equations we rewrite them as follows.

$$v'_1\cos\theta = 2v/\sqrt{2} - v'_2\cos\phi, \quad v'_1\sin\theta = -v'_2\sin\phi. \tag{29}$$

Then, squaring both sides of both equations and adding them gives,

$$v'^2_1 = 2v^2 + v'^2_2 - 2\sqrt{2}vv'_2\cos\phi. \tag{30}$$

Solving for v'^2_1 from the energy conservation equation and substituting in the above equation gives,

$$2v^2 - v'^2_2 = 2v^2 + v'^2_2 - 2\sqrt{2}vv'_2\cos\phi. \tag{31}$$

That is,

$$v'^2_2 - \sqrt{2}vv'_2\cos\phi = 0. \tag{32}$$

This, being a quadratic equation, has two solutions for v'_2. One of them is $v'_2 = 0$. This means the second particle comes to a stop after collision and hence, could not be moving at an angle of 60°. So, this solution is invalid. The other solution gives

$$v'_2 = \sqrt{2}v\cos(60^\circ) = v/\sqrt{2}. \tag{33}$$

Using this in the energy conservation equation gives,

$$v'_1 = \sqrt{2v^2 - v'^2_2} = \sqrt{3/2}\, v. \tag{34}$$

Using this in the momentum conservation equations gives ($\phi = 60^\circ$),

$$\sin\theta = -v'_2\sin\phi/v'_1 = -1/2, \quad \cos\theta = \frac{\sqrt{2}v - (v/\sqrt{2})\cos\phi}{\sqrt{3/2}\, v} = \sqrt{3}/2. \tag{35}$$

Hence,

$$\theta = -30^\circ. \tag{36}$$

Problem 5

If the density of nitrogen at a height h is half that at height zero, then at that height $N(E) = N(0)/2$ for any fixed volume of gas. Using this in the Boltzmann distribution equation gives

$$0.5 = e^{-\beta E} = e^{-\beta mgh}$$

Hence,

$$h = -\frac{kT}{mg}\ln(0.5) = 6300 \text{ m}$$

where we have used the fact that the mass of a nitrogen molecule is its molecular weight (28) times the mass of a proton (1.67×10^{-27} kg)

Problem 6

Let,

$$u = \vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t = k_x x + k_y y + k_z z - \omega t.$$

Then,

$$\frac{\partial u}{\partial x} = k_x, \quad \frac{\partial u}{\partial y} = k_y, \quad \frac{\partial u}{\partial z} = k_z, \quad \frac{\partial u}{\partial t} = -\omega.$$

So, using the chain-rule for derivatives,

$$\frac{\partial \phi}{\partial x} = \frac{d\phi}{du}\frac{\partial u}{\partial x} = k_x \frac{d\phi}{du},$$

and,

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}\left(k_x \frac{d\phi}{du}\right) = k_x \frac{\partial}{\partial x}\frac{d\phi}{du} = k_x \frac{d^2\phi}{du^2}\frac{\partial u}{\partial x} = k_x^2 \frac{d^2\phi}{du^2}.$$

Similarly,

$$\frac{\partial^2 \phi}{\partial y^2} = k_y^2 \frac{d^2\phi}{du^2}, \quad \frac{\partial^2 \phi}{\partial z^2} = k_z^2 \frac{d^2\phi}{du^2}, \quad \frac{\partial^2 \phi}{\partial t^2} = \omega^2 \frac{d^2\phi}{du^2}.$$

Substituting these in the wave equation gives,

$$(k_x^2 + k_y^2 + k_z^2)\frac{d^2\phi}{du^2} - \frac{\omega^2}{v^2}\frac{d^2\phi}{du^2} = 0.$$

If $\frac{d^2\phi}{du^2} \neq 0$, then,

$$k_x^2 + k_y^2 + k_z^2 - \frac{\omega^2}{v^2} = 0.$$

Then,

$$v = \frac{\omega}{k},$$

where,

$$k = \sqrt{k_x^2 + k_y^2 + k_z^2},$$

is the magnitude of the vector $\vec{\mathbf{k}}$.

Problem 9

Part a

$$y = mL\lambda/d = 3 \times 2.0 \times 590 \times 10^{-9}/(0.1 \times 10^{-3}) = 0.035 \text{ m}$$

Part b

$$y = mL\lambda/d = 3 \times 2.0 \times 780 \times 10^{-9}/(0.1 \times 10^{-3}) = 0.047 \text{ m}$$

Part c

$$y = (m + 1/2)L\lambda/d = 2.5 \times 2.0 \times 590 \times 10^{-9}/(0.1 \times 10^{-3}) = 0.030 \text{ m}$$

Part d

$$y = (m + 1/2)L\lambda/d = 2.5 \times 2.0 \times 780 \times 10^{-9}/(0.1 \times 10^{-3}) = 0.039 \text{ m}$$

Problem 10

Part a

$$\sin\theta = m\lambda/d = 2 \times 590 \times 10^{-9}/(5 \times 10^{-6}) = 0.236$$

Hence,

$$\theta = 13.7^\circ$$

Part b

$$\sin\theta = m\lambda/d = 2 \times 780 \times 10^{-9}/(5 \times 10^{-6}) = 0.312$$

Hence,

$$\theta = 18.2^\circ$$

Problem 11

$$\lambda = 2d \sin\theta/m = 2 \times 2.09 \times 10^{-10} \times \sin 30^\circ/1 = 2.09 \times 10^{-10} \text{ m}$$

Problem 15

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

Problem 16

$$R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

Problem 17

$$\begin{aligned} R = R_y(\theta_y)R_x(\theta_x) &= \begin{pmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{pmatrix} \\ &= \begin{pmatrix} \cos\theta_y & \sin\theta_y \sin\theta_x & \sin\theta_y \cos\theta_x \\ 0 & \cos\theta_x & -\sin\theta_x \\ -\sin\theta_y & \cos\theta_y \sin\theta_x & \cos\theta_y \cos\theta_x \end{pmatrix} \end{aligned}$$

Problem 18

Inserting $\theta_x = 180^\circ$ and $\theta_y = 180^\circ$ in the result of the last problem gives

$$R = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos 180^\circ & -\sin 180^\circ & 0 \\ \sin 180^\circ & \cos 180^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix}$$