Solutions
Chapter 9

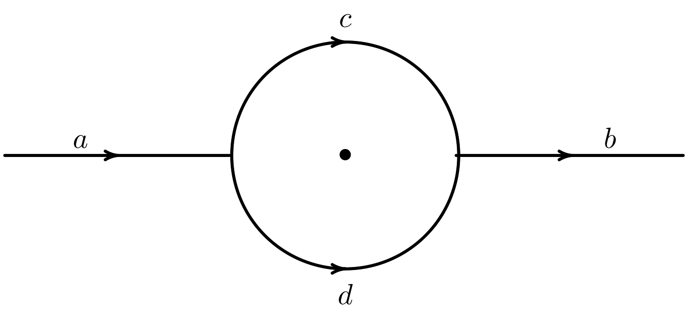
Problem 1

The measured magnetic field at the specified point is zero because the Earth's magnetic field exactly cancels the wire's magnetic field. The magnetic field due to the wire alone can be seen to be horizontal and southwards (right-hand-rule). Hence, the Earth's magnetic field at the same point must be horizontal and northwards.

The magnitude of the Earth's magnetic field must be equal to that of the wire at the specified point and hence, it is

$$B = \frac{\mu_0 i}{2\pi r} = \frac{2 \times 10^{-7} \times 20}{0.10} = 4.0 \times 10^{-5} \text{ T}$$

Problem 2



Biot Savart law gives:

$$d\vec{\mathbf{B}} = \frac{\mu_0 i}{4\pi r^3} d\vec{\mathbf{s}} \times \vec{\mathbf{r}}$$

Along the straight line sections of the wire (labelled a and b), $d\vec{\mathbf{s}}$ (as defined) is along the wire and so is $\vec{\mathbf{r}}$ (as it must point towards the target which is the center of the circle). This gives: $d\vec{\mathbf{s}} \times \vec{\mathbf{r}} = 0$. Hence, the magnetic field due to these sections is zero.

For the upper semicircular section of the wire (labelled c), $d\vec{\mathbf{s}}$ is still along the direction of the current (tangent to the circle). The direction of $\vec{\mathbf{r}}$, being towards the center, is in the radial direction. Hence, $d\vec{\mathbf{s}}$ and $\vec{\mathbf{r}}$ are perpendicular to each other and using the right-hand-rule their cross product is directed into the page. The magnitude of $d\vec{\mathbf{B}}$ due to any infinitesimal section $d\vec{\mathbf{s}}$ of the wire would then be (current is $i/4$):

$$dB = \frac{\mu_0 (i/4)}{4\pi r^3} |d\vec{\mathbf{s}} \times \vec{\mathbf{r}}| = \frac{\mu_0 i}{16\pi r^2} ds$$

as the angle between $d\vec{\mathbf{s}}$ and $\vec{\mathbf{r}}$ is 90 degrees and the magnitude of the vector $\vec{\mathbf{r}}$ is r. Integrating this over the semicircular section (note that r is a constant) one gets:

$$B = \frac{\mu_0 i}{16\pi r^2}\int_0^{\pi r} ds = \frac{\mu_0 i}{16\pi r^2}\pi r = \frac{\mu_0 i}{16 r}$$

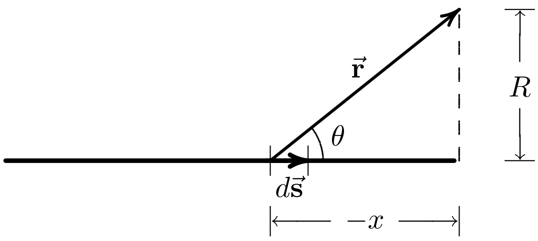
The magnetic field due to the lower semicircular section is found in a similar fashion to be

$$B' = \frac{3\mu_0 i}{16 r}.$$

The direction of this is out of the page (right-hand-rule). So, the total magnetic field is out of the page and its magnitude is

$$B_t = B' - B = \frac{\mu_0 i}{8r}.$$

Problem 3



Magnetic field due to the piece $d\vec{\mathbf{s}}$ is:

$$d\vec{\mathbf{B}} = \frac{\mu_0 i}{4\pi r^3} d\vec{\mathbf{s}} \times \vec{\mathbf{r}}$$

So the direction is out of the page and the magnitude

$$dB = \frac{\mu_0 i}{4\pi r^3}|d\vec{\mathbf{s}} \times \vec{\mathbf{r}}| = \frac{\mu_0 i}{4\pi r^2} ds\, \sin\theta$$

If we choose the wire to be along the x axis with the origin at its *right* end,

$$ds = dx, \quad r = (x^2 + R^2)^{1/2}, \quad \sin\theta = \frac{R}{r} = \frac{R}{(x^2+R^2)^{1/2}}$$

Hence,

$$dB = \frac{\mu_0 i R}{4\pi}\frac{dx}{(x^2+R^2)^{3/2}}$$

Integrating this along the length of the wire (limits from $-L$ to 0) gives:

$$B = \int dB = \frac{\mu_0 i R}{4\pi}\int_{-L}^{0}\frac{dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 i R}{4\pi}\left[\frac{x}{R^2(x^2+R^2)^{1/2}}\right]_{-L}^{0} = \frac{\mu_0 i L}{4\pi R(L^2+R^2)^{1/2}}$$

Problem 4

The points where the magnetic fields due to the two wires cancel to give zero will all lie on a line parallel to the two wires and between them. Let the distance of this line from the wire with current i be x. On this line the magnetic fields due to the two wires are oppositely directed and equal in magnitude. Hence,

$$\frac{\mu_0 i}{2\pi x} = \frac{\mu_0 (2i)}{2\pi (d - x)}$$

Solving for x gives

$$x = d/3.$$

Problem 5

The gravitational force is downwards and of magnitude mg. To balance the wire, the magnetic force must be equal in magnitude but opposite in direction to the gravitational force. Hence,

$$mg = \frac{\mu_0 i i_0 L}{2\pi d}.$$

Solving for i gives

$$i = \frac{2\pi d m g}{\mu_0 i_0 L}.$$

Problem 6

Part a

As the direction of integration is clockwise, positive currents are into the page. So, using Ampere's law

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0(-i - i) = -2\mu_0 i$$

Part b

As the direction of integration is counterclockwise, positive currents are out of the page. So, using Ampere's law

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0(i + i) = 2\mu_0 i$$

Part c

As the direction of integration is counterclockwise, positive currents are out of the page. So, using Ampere's law

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0(i - i) = 0$$

Part d

As the direction of integration is clockwise, positive currents are into the page. So, using Ampere's law

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0(i - i) = 0$$

Problem 7

$$B = \mu_0 n i = 4\pi \times 10^{-7} \times \frac{5000}{2.00} \times 3.00 = 9.42 \times 10^{-3}\ \mathrm{T}$$