Solutions
Chapter 6

Problem 1

Let the current be i (clockwise). Then using the loop rule gives

$$-4.0i + 100 - 10i - 50 = 0$$

Solving for i gives

$$i = 50/14 = 3.6 \text{ A}$$

Then, adding voltages from point P to point P_1 gives

$$V_{P_1} = V_P - 50 = 50 - 50 = 0V$$

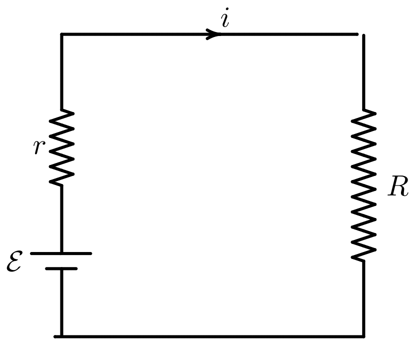
Similarly, for point P_2

$$V_{P_2} = V_P - 50 - 4.0i = -14 \text{ V}$$

Similarly, for point P_3

$$V_{P_3} = V_P + 10i = 86 \text{ V}$$

Problem 2



Let the resistance of the voltmeter be R and the internal resiatance of the battery be r. Then the voltage V across the battery is also the voltage across the voltmeter:

$$V = \mathcal{E} - ir = iR.$$

This follows from the loop equation

$$\mathcal{E} - ir - iR = 0.$$

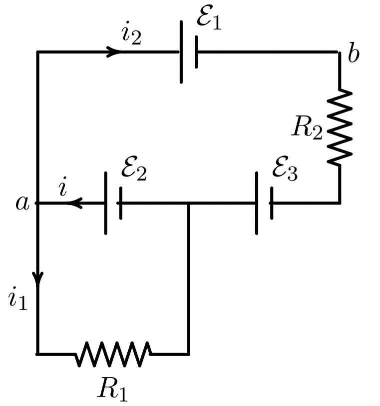
From the first equation we get

$$i = V/R,$$

and

$$r = \frac{\mathcal{E} - V}{i} = \frac{(\mathcal{E} - V)R}{V} = \frac{(12 - 1.5) \times 4.0 \times 10^5}{1.5} = 2.8 \times 10^6 \Omega$$

Problem 3



Choosing the current directions as shown, the junction rule gives

$$i = i_1 + i_2$$

The loop rule applied to the two loops gives

$$-\mathcal{E}_1 - i_2 R_2 + \mathcal{E}_3 + \mathcal{E}_2 = 0.$$

$$-\mathcal{E}_2 + i_1 R_1 = 0.$$

Solving the last two equations gives

$$i_2 = \frac{\mathcal{E}_2 + \mathcal{E}_3 - \mathcal{E}_1}{R_2} = \frac{4.0 + 2.0 - 5.0}{5.0} = 0.20 \text{ A}.$$

$$i_1 = \frac{\mathcal{E}_2}{R_1} = 0.40 \text{ A}.$$

Then from the first equation

$$i = 0.40 + 0.20 = 0.60 \text{ A}.$$

By adding voltages from point b to point a we get

$$V_a = V_b - i_2 R_2 + \mathcal{E}_3 + \mathcal{E}_2.$$

Hence,

$$V_a - V_b = -i_2 R_2 + \mathcal{E}_3 + \mathcal{E}_2 = 5.0 \text{ V}.$$

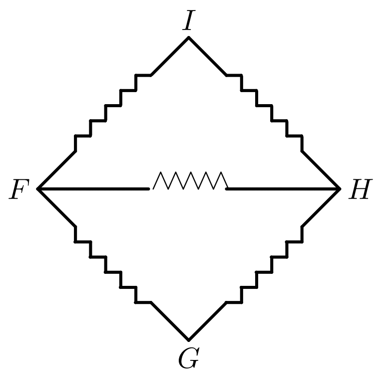
One could take an alternate path and get

$$V_a = V_b + \mathcal{E}_1.$$

This would also give

$$V_a - V_b = \mathcal{E}_1 = 5.0 \text{ V}.$$

Problem 4



Part a

Resistance in the path FIH is

$$R_1 = R + R = 2R$$

Resistance in the path FGH is

$$R_2 = R + R = 2R$$

So between points F and H there are three resistances in parallel ($2R$, R and $2R$). Hence, the net resistance R_{FH} is given by

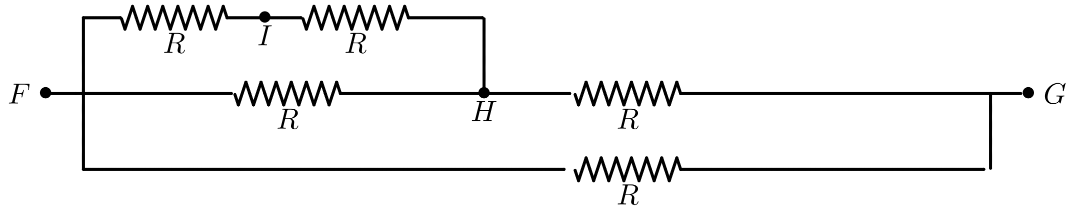
$$\frac{1}{R_{FH}} = \frac{1}{2R} + \frac{1}{R} + \frac{1}{2R} = \frac{2}{R}$$

Hence,

$$R_{FH} = \frac{R}{2}.$$

Part b

It helps to redraw the circuit as follows.



The resistance of the direct path from F to H is in parallel to the path FIH. So its net resistance is R_3 such that

$$\frac{1}{R_3} = \frac{1}{R_1} + \frac{1}{R} = \frac{1}{2R} + \frac{1}{R} = \frac{3}{2R}.$$

Hence,

$$R_3 = \frac{2R}{3}.$$

R_3 is in series with the branch GH. So their net resistance is

$$R_4 = R_3 + R = \frac{2R}{3} + R = \frac{5R}{3}$$

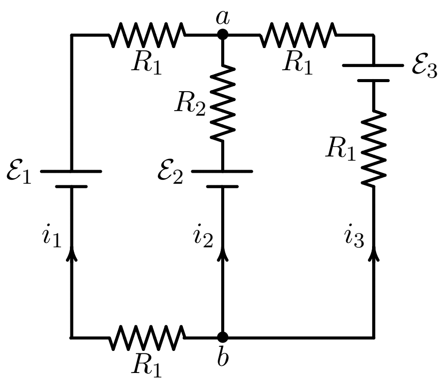
R_4 is in parallel to the bottom branch going directly from F to G. So the net resistance between F and G is R_{FG} such that

$$\frac{1}{R_{FG}} = \frac{1}{R_4} + \frac{1}{R} = \frac{3}{5R} + \frac{1}{R} = \frac{8}{5R}$$

Hence,

$$R_{FG} = \frac{5R}{8}.$$

Problem 5



Part a

Assume current directions as shown in the figure. The junction equation for junction a is

$$i_1 + i_2 + i_3 = 0$$

The loop equations for the two loops are

$$\mathcal{E}_1 - i_1 R_1 + i_2 R_2 - \mathcal{E}_2 - i_1 R_1 = 0$$

$$\mathcal{E}_2 - i_2 R_2 + i_3 R_1 - \mathcal{E}_3 + i_3 R_1 = 0$$

So the three equations to be solved are

$$\begin{aligned} i_1 + i_2 + i_3 &= 0 \\ -10 i_1 + 2 i_2 + 4 &= 0 \\ -2 i_2 + 10 i_3 + 2 &= 0 \end{aligned}$$

These can be reduced to

$$i_1 + i_2 + i_3 = 0 \quad (1)$$

$$-5 i_1 + i_2 + 2 = 0 \quad (2)$$

$$-i_2 + 5 i_3 + 1 = 0 \quad (3)$$

Using equations 2 and 3 gives

$$i_1 = (i_2 + 2)/5 \tag{4}$$
$$i_3 = (i_2 - 1)/5 \tag{5}$$

Using equations 4 and 5 in equation 1 and solving for i_2 gives

$$i_2 = -1/7 = -0.14 \text{ A}$$

Then using equation 4 gives

$$i_1 = 13/35 = 0.37 \text{ A}$$

And using equation 5 gives

$$i_3 = -8/35 = -0.23 \text{ A}$$

Part b

Adding voltages from point b to point a gives

$$V_a = V_b + \mathcal{E}_2 - i_2 R_2$$

So

$$V_a - V_b = \mathcal{E}_2 - i_2 R_2 = 4.3 \text{ V}$$