Solutions
Chapter 4

Problem 1

Potential energy difference is

$$\Delta U = q\Delta V = 1.60 \times 10^{-19} \times 1.5 \times 10^9 \text{ J} = 2.4 \times 10^{-10} \text{ J} = \frac{2.4 \times 10^{-10}}{1.60 \times 10^{-19}} \text{ eV} = 1.5 \times 10^9 \text{ eV}$$

Problem 2

The potential difference is

$$\Delta V = -\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int \frac{\sigma}{2\epsilon_0} ds(-1) = \frac{\sigma}{2\epsilon_0} s$$

If the separation between equipotentials is s, then $\Delta V = 75$ V and

$$s = \frac{2\Delta V \epsilon_0}{\sigma} = 2.7 \times 10^{-3} \text{ m}$$

Problem 3

Part a

$$\Delta V = Ed = 2.00 \times 10^4 \times 0.010 = 200 \text{ V}$$

Part b

The kinetic energy gained by the electron will be its potential energy lost. Hence,

$$mv^2/2 = e\Delta V.$$

Solving for v gives

$$v = \sqrt{\frac{2e\Delta V}{m}} = 8.38 \times 10^6 \text{ m/s}$$

Problem 4

With infinity as reference, the potential is

$$V(r) = -\int_{\infty}^{r} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_{\infty}^{r} E\, dr' = -\int_{\infty}^{r} E_0 e^{-r'/r_0} dr' = E_0 r_0 e^{-r/r_0}.$$

Problem 5

The potential difference between any two points 'a' and 'b' is defined to be:

$$V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Let the reference be at 'a' a distance r_a from the center. Then $V_a = V(r_a) = 0$. Let 'b' be at any arbitrary distance r from the center. Then $V(r) = V_b$. Now, if the integration path is chosen along a radial line:

$$V(r) = -\int_{r_a}^{r} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_{r_a}^{r} E\, dr'$$

Part a

If the center is chosen as reference, then $r_a = 0$. If $r < R$ then the complete path of integration is inside the sphere. Hence, the corresponding formula for E is used to compute $V(r)$ as follows.

$$V(r) = -\int_0^r \frac{qr'}{4\pi\epsilon_0 R^3} dr' = -\frac{qr^2}{8\pi\epsilon_0 R^3}$$

Part b

For this part we notice that part of the path of integration is inside the sphere and part of it is outside. Hence, we need to split up the integral into the two parts and use the appropriate formula for E in each part. This gives

$$V(r) = -\int_0^R \frac{qr'}{4\pi\epsilon_0 R^3} dr' - \int_R^r \frac{q}{4\pi\epsilon_0 r'^2} dr' = -\frac{q}{8\pi\epsilon_0 R} - \frac{q}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right) = -\frac{q}{4\pi\epsilon_0}\left(\frac{3}{2R} - \frac{1}{r}\right)$$

Part c

If the reference is at infinity and $r < R$, then the path of integration is once again split between the two regions. So,

$$V(r) = -\int_{\infty}^{r} E\, dr' = -\int_{\infty}^{R} \frac{q}{4\pi\epsilon_0 r'^2} dr' - \int_R^r \frac{qr'}{4\pi\epsilon_0 R^3} dr' = \frac{q}{4\pi\epsilon_0 R} - \frac{q(r^2 - R^2)}{8\pi\epsilon_0 R^3} = \frac{q(3R^2 - r^2)}{8\pi\epsilon_0 R^3}.$$

Part d

In this part the path of integration is completely outside the sphere. Hence,

$$V(r) = -\int_{\infty}^{r} E\,dr' = -\int_{\infty}^{r} \frac{q}{4\pi\epsilon_0 r'^2} dr' = \frac{q}{4\pi\epsilon_0 r}.$$

Problem 6

Part a

$$V_B - V_A = \frac{q}{4\pi\epsilon_0 d_B} - \frac{q}{4\pi\epsilon_0 d_A} = 1.1 \times 10^4 \text{ V}$$

Part b

As the distances are the same the answer is the same as in part a.

Problem 7

The potential is

$$V = V_1 + V_2 + V_3 = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3},$$

where r_1, r_2 and r_3 are the respective distances of the three charges from the point P. Hence,

$$r_1 = r_3 = \sqrt{d^2 + d^2} = \sqrt{2}d, \quad \text{and} \quad r_2 = d$$

Hence,

$$V = \frac{kq}{\sqrt{2}d} + \frac{-2kq}{d} + \frac{kq}{\sqrt{2}d} = \frac{kq}{d}\left(\frac{1}{\sqrt{2}} - 2 + \frac{1}{\sqrt{2}}\right) = -(2 - \sqrt{2})\frac{kq}{d}.$$

Problem 8

The potential is

$$V = V_1 + V_2 + V_3 + V_4 + V_5 = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} + \frac{kq_4}{r_4} + \frac{kq_5}{r_5},$$

where r_1, r_2, r_3, r_4 and r_5 are the respective distances of the five charges from the origin. Hence,

$$r_1 = r_3 = r_4 = 2d, \quad r_2 = 3d, \text{ and } r_5 = \sqrt{9d^2 + 4d^2} = \sqrt{13}d.$$

So, using the given values for the charges,

$$V = \frac{kq}{2d} + \frac{-2kq}{3d} + \frac{-kq}{2d} + \frac{3kq}{2d} + \frac{-3kq}{\sqrt{13}d} = \frac{kq}{d}\left(\frac{5}{6} - \frac{3}{\sqrt{13}}\right)$$

Problem 9

Potential is zero a distance x from the origin if (for positive x)

$$\frac{q_1}{4\pi\epsilon_0 x} + \frac{q_2}{4\pi\epsilon_0 (d-x)} = 0$$

Replacing $q_1 = q$ and $q_2 = -2q$ gives

$$\frac{q}{4\pi\epsilon_0 x} - \frac{2q}{4\pi\epsilon_0 (d-x)} = 0$$

or

$$\frac{1}{x} = \frac{2}{d-x}$$

Solving this for x gives

$$x = d/3$$

For negative x values

$$-\frac{q_1}{4\pi\epsilon_0 x} + \frac{q_2}{4\pi\epsilon_0 (d-x)} = 0$$

Then

$$-\frac{q}{4\pi\epsilon_0 x} - \frac{2q}{4\pi\epsilon_0 (d-x)} = 0$$

Solving for x gives

$$x = -d$$

Problem 10

Potential due to an element of charge dq is

$$dV = \frac{dq}{4\pi\epsilon_0 r}$$

For point P

$$r = (R^2 + z^2)^{1/2}$$

So

$$dV = \frac{dq}{4\pi\epsilon_0 \sqrt{R^2 + z^2}}$$

As R and z are both constants for all points on the circle, integrating dV gives

$$V = \frac{1}{4\pi\epsilon_0\sqrt{R^2+z^2}}\int dq$$

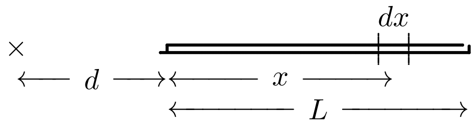
Now,

$$\int dq = 2Q - Q = Q.$$

Hence,

$$V = \frac{Q}{4\pi\epsilon_0\sqrt{R^2+z^2}}$$

Problem 11



The charge on the element of length dx is $dq = \lambda\, dx$. So the potential due to it is

$$dV = \frac{dq}{4\pi\epsilon_0 r} = \frac{\lambda\, dx}{4\pi\epsilon_0(x+d)} = \frac{c(L-x)\, dx}{4\pi\epsilon_0(x+d)}$$

Integrating this over the full length gives

$$V = \int_0^L \frac{c(L-x)\, dx}{4\pi\epsilon_0(x+d)} = \frac{c}{4\pi\epsilon_0}\int_0^L \frac{(L-x)\, dx}{x+d} = \frac{c}{4\pi\epsilon_0}\left[(L+d)\ln\left(\frac{L+d}{d}\right) - L\right]$$

Problem 12

Part a

The potential along the z axis was found to be

$$V = \frac{Q}{4\pi\epsilon_0\sqrt{R^2+z^2}}$$

So, the z component of the electric field is:

$$E_z = -\frac{\partial V}{\partial z} = -\frac{Q}{4\pi\epsilon_0}(-1/2)(R^2+z^2)^{-3/2}2z = \frac{Qz}{4\pi\epsilon_0(R^2+z^2)^{3/2}}$$

Part b

The x component cannot be found as the given expression for potential is valid only along the z axis. This means x has been set to zero. To find the x component of the electric field one has to find the derivative of V with respect to x. This cannot be done when x has already been set to zero. In other words, we need to know the functional dependence of V on x to find the x component.

Problem 13

$$E_x = -\frac{\partial V}{\partial x} = -10x - 12x^2\sqrt{z}$$

$$E_y = -\frac{\partial V}{\partial y} = -20e^{-y}$$

$$E_z = -\frac{\partial V}{\partial z} = -2x^3/\sqrt{z}$$