Solutions
Chapter 3

Problem 1

Part a

The flux through a surface with constant electric field is

$$\Phi = \vec{\mathbf{E}} \cdot \Delta\vec{\mathbf{A}}$$

For the front face $\Delta\vec{\mathbf{A}} = a^2\hat{\mathbf{i}}$, where a is the length of a side of the cube. Hence,

$$\Phi = (5.00\hat{\mathbf{k}}) \cdot (a^2\hat{\mathbf{i}}) = 0$$

Part b

Similarly,

$$\Phi = (5.00\hat{\mathbf{j}}) \cdot (a^2\hat{\mathbf{j}}) = 5.00a^2 = 20.0 \text{ Nm}^2/\text{C}$$

Part c

Similarly,

$$\Phi = (5.00\hat{\mathbf{k}}) \cdot (a^2\hat{\mathbf{k}}) = 5.00a^2 = 20.0 \text{ Nm}^2/\text{C}$$

Part d

Similarly,

$$\Phi = (2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}} - 5.00\hat{\mathbf{k}}) \cdot (a^2\hat{\mathbf{i}}) = 2.00a^2 = 8.00 \text{ Nm}^2/\text{C}$$

Part e

For the six faces $\Delta\vec{\mathbf{A}}$ is:

$a^2\hat{\mathbf{i}}$ (front), $-a^2\hat{\mathbf{i}}$ (rear), $a^2\hat{\mathbf{j}}$ (right), $-a^2\hat{\mathbf{j}}$ (left), $a^2\hat{\mathbf{k}}$ (top), $-a^2\hat{\mathbf{k}}$ (bottom).

So, if $\vec{\mathbf{E}} = 2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}} - 5.00\hat{\mathbf{k}}$, total flux is

$$\Phi = (2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}} - 5.00\hat{\mathbf{k}}) \cdot (a^2\hat{\mathbf{i}}) + (2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}} - 5.00\hat{\mathbf{k}}) \cdot (-a^2\hat{\mathbf{i}}) + (2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}} - 5.00\hat{\mathbf{k}}) \cdot (a^2\hat{\mathbf{j}})$$
$$+(2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}} - 5.00\hat{\mathbf{k}}) \cdot (-a^2\hat{\mathbf{j}}) + (2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}} - 5.00\hat{\mathbf{k}}) \cdot (a^2\hat{\mathbf{k}}) + (2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}} - 5.00\hat{\mathbf{k}}) \cdot (-a^2\hat{\mathbf{k}}) = 0$$

Problem 2

Irrespective of the shape of the closed surface, Gauss' law gives the total flux to be

$$\Phi = \frac{q}{\epsilon_0} = \frac{2.00 \times 10^{-6}}{8.85 \times 10^{-12}} = 2.26 \times 10^5 \text{ Nm}^2/\text{C}$$

Problem 3

Part a

The $\Delta\vec{\mathbf{A}}$ for the six faces is the same as before. $\vec{\mathbf{E}}$ is variable on the faces perpendicular to the $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ directions. However, as $\vec{\mathbf{E}}$ is perpendicular to $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$, these faces do not contribute to Φ. So, we can still use the constant electric field form of the flux formula. As $x = 0$ on the rear face, the electric field on it ($\vec{\mathbf{E}} = 2.00x\hat{\mathbf{i}}$) is also zero. Hence, the only contribution to the total flux comes from the front face where $\vec{\mathbf{E}} = 2.00x\hat{\mathbf{i}} = 2.00a\hat{\mathbf{i}}$. So,

$$\Phi = 2.00a\hat{\mathbf{i}} \cdot (a^2\hat{\mathbf{i}}) = 2.00a^3 = 2.00 \times (2.00)^3 = 16.0 \text{ Nm}^2/\text{C}$$

Then, using Gauss' law we get the enclosed charge to be

$$q = \epsilon_0 \Phi = 1.42 \times 10^{-10} \text{ C}$$

Part b

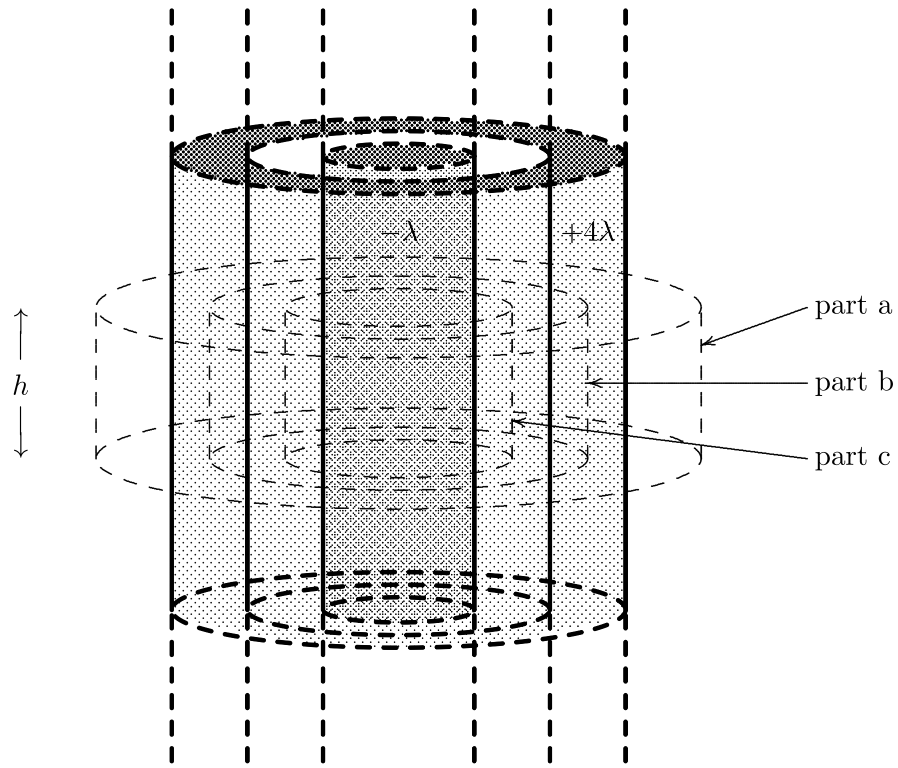
It can be seen that constant electric fields always produce a zero flux over a closed surface. Hence, only the variable part of $\vec{\mathbf{E}}$ will contribute. The variable part being the same as in part a, we once again get

$$\Phi = 16.0 \text{ Nm}^2/\text{C}$$

Then, using Gauss' law we get the enclosed charge to be

$$q = \epsilon_0 \Phi = 1.42 \times 10^{-10} \text{ C}$$

Problem 4



Part a

The figure above shows the cylindrical Gaussian surface to be used for this part (height h, radius r). The flux through it is

$$\Phi = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 2\pi r h E$$

The charge density on the inner cylinder is $-\lambda$. The charge density on the shell is 4λ. So the total charge inside the Gaussian surface shown is $(4\lambda - \lambda)h = 3\lambda h$. Hence, using Gauss' law,

$$\Phi = 2\pi r h E = 3\lambda h/\epsilon_0$$

Solving for E gives

$$E = \frac{3\lambda}{2\pi\epsilon_0 r}$$

Part b

Due to Gauss' law, it is known that all charges on a conductor remain on the surface under static conditions. So, the charges on the shell will be found only on its inner and outer surfaces. Let the charge density on the inner surface of the shell be λ_i. Then, using the Gaussian surface shown (between the inner and outer surfaces of the shell), we see that the charge inside it is $(-\lambda + \lambda_i)h$. The electric field on this Gaussian surface is zero as it is inside a conductor. Hence,

$$\Phi = 0 = \frac{(-\lambda + \lambda_i)h}{\epsilon_0}$$

This gives

$$\lambda_i = \lambda$$

As the total charge density on the shell is $+4\lambda$ and the charges are only on the surface, the outer surface must have a charge density of 3λ.

Part c

For the Gaussian surface between the cylinder and the shell (height h can be the same as in part a, but radius r is clearly smaller), the charge inside is $-\lambda h$. So, using Gauss' law,

$$\Phi = 2\pi r h E = -\lambda h/\epsilon_0$$

Solving for E gives

$$E = -\frac{\lambda}{2\pi\epsilon_0 r}$$

The negative sign indicates that the electric field is inwards.

Problem 5

Part a

The electric field magnitude due to each sheet is $\sigma/(2\epsilon_0)$. Above the two sheets the electric fields due to both are upwards and hence, the total electric field is

$$E = 2 \times \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Part b

Between the two sheets the electric fields are oppositely directed. So they give a total of $E = 0$.

Part c

Below the two sheets both contributions are downwards. So they add to give a magnitude of

$$E = 2 \times \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Problem 6

The electric field magnitude due to each plate is $\sigma/(2\epsilon_0)$. Between the plates, the electric fields due to both of them point towards the negative plate. Hence, the total field has a magnitude $E = \sigma/\epsilon_0$. Its direction is towards the negative plate. The force on the positive charge is $F = qE$. Hence from Newton's second law

$$ma = qE,$$

which gives the acceleration to be

$$a = \frac{qE}{m} = \frac{q\sigma}{m\epsilon_0}.$$

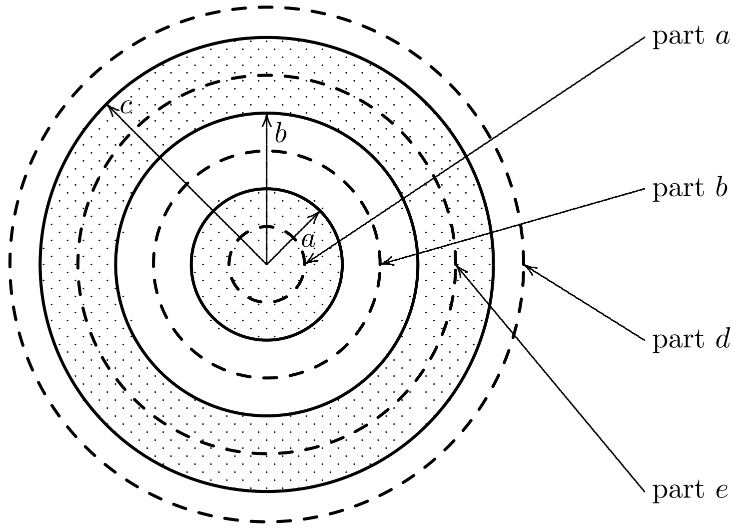
For uniformly accelerated motion from the positive plate to the negative plate, the distance travelled will be

$$d = at^2/2.$$

Hence, the time taken is

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2dm\epsilon_0}{q\sigma}}.$$

Problem 7



From symmetry the electric field is known to be radially directed. Also according to the symmetry the Gaussian surface is chosen to be spherical. The figure shows the Gaussian surface chosen for each part of this problem. The radius of the surface in each case is r. The flux through the surface in each case is computed as follows:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\, dA = E \oint dA = E4\pi r^2$$

as E is a constant on the spherical surface.

Part a

In the region where $r < a$, the Gaussian surface has a charge of qr^3/a^3 within it. Hence, from Gauss' law:

$$E4\pi r^2 = \frac{qr^3}{\epsilon_0 a^3}$$

which gives:

$$E = \frac{qr}{4\pi\epsilon_0 a^3}$$

Part b

For $a < r < b$, the total charge inside the Gaussian surface is q. Hence, from Gauss' law:

$$E4\pi r^2 = \frac{q}{\epsilon_0}$$

which gives:

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

Part c

The shell being conducting, $E = 0$ within it.

Part d

For $r > c$, the total charge inside the Gaussian surface is the sum of the charges on the inner sphere and the spherical shell which is: $q + 2q = 3q$. Hence, from Gauss' law:

$$E4\pi r^2 = \frac{3q}{\epsilon_0}$$

which gives:

$$E = \frac{3q}{4\pi\epsilon_0 r^2}$$

Part e

As the shell is a conductor, a Gaussian surface within it will have $E = 0$ on it. So,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E4\pi r^2 = 0$$

Hence, from Gauss' law it is seen that the total charge within this Gaussian surface is zero. So, if q_i is the charge on the inner surface of the shell,

$$q_i + q = 0.$$

This gives

$$q_i = -q.$$

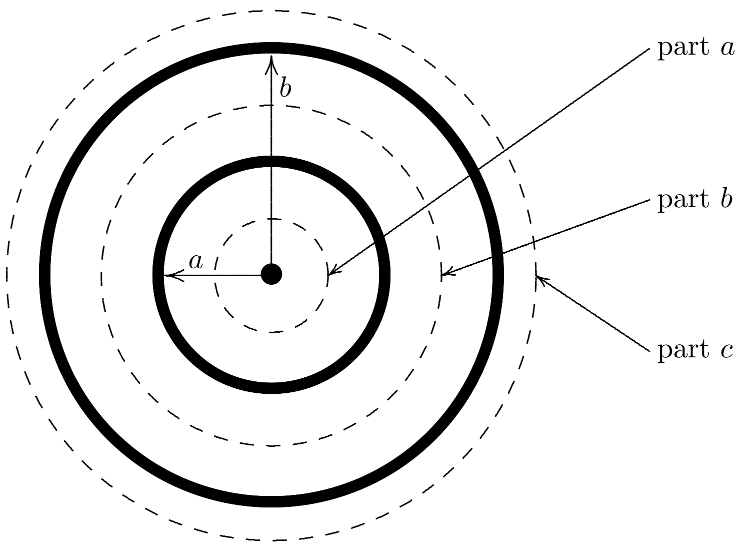
If the charge on the outer surface of the shell is q_o, then

$$q_i + q_o = 2q,$$

as $2q$ is the total charge on the shell and all the charge on a conductor stays on its surface. This gives

$$q_o = 2q - q_i = 3q.$$

Problem 8



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$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\, dA = E \oint dA = E4\pi r^2$$

as E is a constant on the spherical surface.

Part a

In the region where $r < a$, the Gaussian surface has a charge of q within it. Hence, from Gauss' law:

$$E4\pi r^2 = \frac{q}{\epsilon_0}$$

which gives:

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

Part b

For $a < r < b$, the total charge inside the Gaussian surface is: $q - 2q = -q$. Hence, from Gauss' law:

$$E4\pi r^2 = \frac{-q}{\epsilon_0}$$

which gives:

$$E = \frac{-q}{4\pi\epsilon_0 r^2}$$

The negative value of E means the field is directed inwards.

Part c

For $r > b$, the total charge inside the Gaussian surface is: $4q + q - 2q = 3q$. Hence, from Gauss' law:

$$E4\pi r^2 = \frac{3q}{\epsilon_0}$$

which gives:

$$E = \frac{3q}{4\pi\epsilon_0 r^2}$$