Solutions
Chapter 2

Problem 1

$$E = \frac{k|q|}{r^2}$$

Hence,

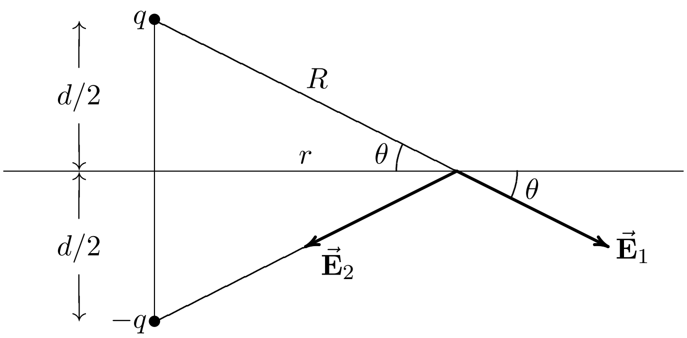
$$r = \sqrt{\frac{k|q|}{E}} = \sqrt{\frac{8.99 \times 10^9 \times 5.0 \times 10^{-6}}{2.0}} = 150 \text{ m}$$

Problem 2

$$\vec{\mathbf{E}} = \frac{kq}{a^2}(-\hat{\mathbf{j}}) + \frac{kq}{2a^2}(\cos 45^\circ \hat{\mathbf{i}} + \sin 45^\circ \hat{\mathbf{j}}) + \frac{k(2q)}{a^2}\hat{\mathbf{i}} =$$

$$= \frac{kq}{2a^2}(\cos 45^\circ + 4)\hat{\mathbf{i}} + \frac{kq}{2a^2}(-2 + \sin 45^\circ)\hat{\mathbf{j}} = 2.6 \times 10^7 \hat{\mathbf{i}} - 7.3 \times 10^6 \hat{\mathbf{j}} \text{ N/C}$$

Problem 3



The electric field due to the positive charge is (R and θ shown in figure)

$$\vec{\mathbf{E}}_1 = \frac{kq}{R^2}(\cos\theta \hat{\mathbf{i}} - \sin\theta \hat{\mathbf{j}})$$

The electric field due to the negative charge is

$$\vec{\mathbf{E}}_2 = \frac{kq}{R^2}(-\cos\theta \hat{\mathbf{i}} - \sin\theta \hat{\mathbf{j}})$$

So the total electric field is

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = -\frac{2kq}{R^2}\sin\theta\hat{\mathbf{j}}$$

As $\sin\theta = d/(2R)$, this gives

$$\vec{\mathbf{E}} = -\frac{2kq}{R^2}\frac{d}{2R}\hat{\mathbf{j}} = -\frac{kqd}{R^3}\hat{\mathbf{j}}$$

Using the Pythagorean theorem $R = \left(r^2 + (d/2)^2\right)^{1/2}$. Hence,

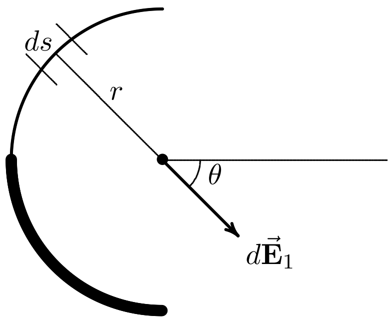
$$\vec{\mathbf{E}} = -\frac{kqd}{\left(r^2 + (d/2)^2\right)^{3/2}}\hat{\mathbf{j}}$$

If r is much larger than d ($r \gg d$), then

$$\vec{\mathbf{E}} = -\frac{kqd}{r^3}\hat{\mathbf{j}} = -\frac{kp}{r^3}\hat{\mathbf{j}} = -\frac{k\vec{\mathbf{p}}}{r^3}$$

The last step uses the fact that the direction of the dipole moment is along the $\hat{\mathbf{j}}$ direction. Hence $\vec{\mathbf{p}} = p\hat{\mathbf{j}}$.

Problem 4



For the top quarter circle, the charge density is $\lambda_1 = 2q/(\pi r)$. The electric field due to the element ds is $d\vec{\mathbf{E}}_1$. The x and y components of this field are:

$$dE_{1x} = dE_1\cos\theta = \frac{k\lambda_1\,ds}{r^2}\cos\theta$$

$$dE_{1y} = dE_1\sin\theta = \frac{k\lambda_1\,ds}{r^2}\sin\theta$$

From the definition of an angle in radians $ds = r\,d\theta$. Hence,

$$dE_{1x} = \frac{k\lambda_1}{r}\cos\theta\,d\theta, \quad dE_{1y} = \frac{k\lambda_1}{r}\sin\theta\,d\theta$$

Integration leads to:

$$E_{1x} = \int dE_{1x} = \frac{k\lambda_1}{r}\int_{-\pi/2}^{0}\cos\theta\,d\theta = \frac{k\lambda_1}{r}$$

$$E_{1y} = \int dE_{1y} = \frac{k\lambda_1}{r} \int_{-\pi/2}^{0} \sin\theta \, d\theta = -\frac{k\lambda_1}{r}$$

The computation of electric field due to the bottom quarter circle is similar to that of the top. The differences are the charge density, which is $\lambda_2 = 4q/(\pi r)$, and the limits of integration, which are 0 and $\pi/2$. This gives:

$$E_{2x} = \int dE_{2x} = \frac{k\lambda_2}{r} \int_{0}^{\pi/2} \cos\theta \, d\theta = \frac{k\lambda_2}{r}$$

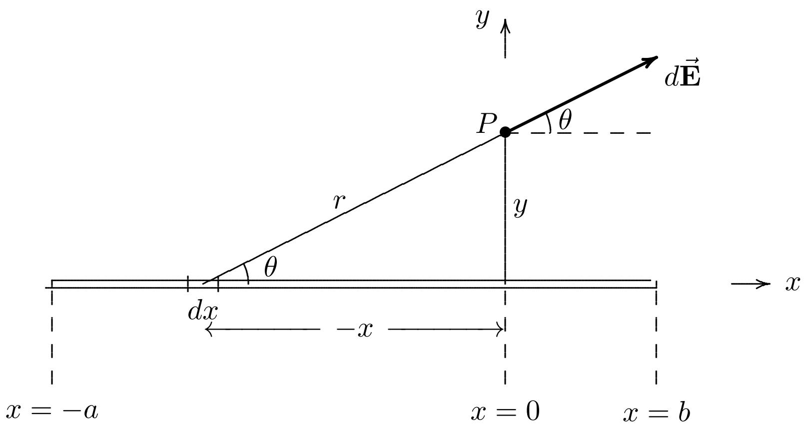
$$E_{2y} = \int dE_{2y} = \frac{k\lambda_2}{r} \int_{0}^{\pi/2} \sin\theta \, d\theta = \frac{k\lambda_2}{r}$$

Now the total electric field due to both quarter circles can be computed:

$$E_x = E_{1x} + E_{2x} = \frac{k\lambda_1}{r} + \frac{k\lambda_2}{r} = \frac{6kq}{\pi r^2}$$

$$E_y = E_{1y} + E_{2y} = -\frac{k\lambda_1}{r} + \frac{k\lambda_2}{r} = \frac{2kq}{\pi r^2}$$

Problem 5



In the figure above the origin of a xy coordinate system is chosen on the rod a distance a from its left end. The rod is along the x axis and the point P is on the y axis. For E_x, the x component of the electric field, we see that an element of rod of length dx produces the field

$$dE_x = dE\,\cos\theta = \frac{k\lambda\,dx}{r^2}\cos\theta = \frac{k\lambda\,dx}{r^2}\left(\frac{-x}{r}\right) = -k\lambda x\frac{dx}{r^3} = -\frac{k\lambda x\,dx}{(y^2+x^2)^{3/2}}$$

Integrating this for the full length of the rod (from $x = -a$ to $x = b$) gives:

$$E_x = \int dE_x = -k\lambda \int_{-a}^{b} \frac{x\;dx}{(y^2+x^2)^{3/2}} = k\lambda\left[\frac{1}{\sqrt{y^2+x^2}}\right]_{x=-a}^{x=b} = k\lambda\left[\frac{1}{\sqrt{y^2+b^2}} - \frac{1}{\sqrt{y^2+a^2}}\right]$$

For E_y, the y component of the electric field, we see that an element of rod of length dx produces the field

$$dE_y = dE\,\sin\theta = \frac{k\lambda\,dx}{r^2}\sin\theta = \frac{k\lambda\,dx}{r^2}\frac{y}{r} = k\lambda y\frac{dx}{r^3} = \frac{k\lambda y\,dx}{(y^2+x^2)^{3/2}}$$

Integrating this for the full length of the rod (from $x = -a$ to $x = b$) gives:

$$E_y = \int dE_y = k\lambda y \int_{-a}^{b} \frac{dx}{(y^2 + x^2)^{3/2}} = \frac{k\lambda y}{y^2} \left[\frac{x}{\sqrt{y^2 + x^2}} \right]_{x=-a}^{x=b} = \frac{k\lambda}{y} \left[\frac{b}{\sqrt{y^2 + b^2}} + \frac{a}{\sqrt{y^2 + a^2}} \right]$$

Problem 6

The force on a charge q due to an electric field $\vec{\mathbf{E}}$ is

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

From Newton's second law of motion $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$. Hence,

$$m\vec{\mathbf{a}} = q\vec{\mathbf{E}}$$

Hence,

$$\vec{\mathbf{E}} = \frac{m\vec{\mathbf{a}}}{q} = \frac{9.11 \times 10^{-31}}{(-1.60 \times 10^{-19})} \times 2.0 \times 10^9 \hat{\mathbf{i}} = -1.1 \times 10^{-2} \hat{\mathbf{i}} \text{ N/C}$$

Problem 7

Part a

Acceleration of the proton is

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{1.60 \times 10^{-19} \times 4.00 \times 10^4}{1.67 \times 10^{-27}} = 3.83 \times 10^{12} \text{ m/s}^2$$

Part b

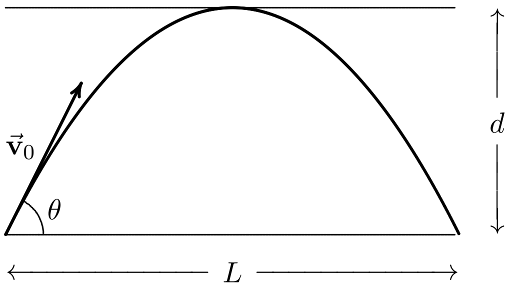
If the proton starts at a speed of zero and ends up with a speed v then, using one of the equations for constant acceleration we get

$$v^2 = 2ax$$

where x is the displacement. Then

$$v = \sqrt{2ax} = \sqrt{2 \times 3.83 \times 10^{12} \times 0.005} = 1.96 \times 10^5 \text{ m/s}$$

Problem 8



Part a

The initial velocity components are:

$$v_{0x} = v_0 \cos\theta$$

$$v_{0y} = v_0 \sin\theta$$

The acceleration is due to an electric force and it is in the downward (negative y) direction. Its value is:

$$a_y = \frac{F_y}{m} = \frac{qE_y}{m} = \frac{-1.60 \times 10^{-19} \times 2000}{9.11 \times 10^{-31}} = -3.51 \times 10^{14}\ \mathrm{m/s^2}$$

At any point of the trajectory, the y displacement and y component of velocity (v_y) are related as follows (constant acceleration mechanics):

$$2a_y y = v_y^2 - v_{0y}^2 = v_y^2 - (v_0 \sin\theta)^2$$

For the maximum θ for which the electron does not hit the upper plate, the highest point of the trajectory must have $y = d$ and at this point $v_y = 0$. Hence,

$$2a_y d = -(v_0 \sin\theta)^2$$

and

$$\sin\theta = \sqrt{-\frac{2a_y d}{v_0^2}} = 0.749$$

So,

$$\theta = 48.5^\circ$$

Part b

The time t taken by the electron to return to the level of the lower plate can be found using the y direction equation of motion:

$$y = v_{0y}t + a_y t^2/2.$$

The lower plate being at $y = 0$, this gives

$$0 = v_{0y}t + a_y t^2/2.$$

This gives two solutions for t. The $t = 0$ solution gives the starting point. Hence, we need the other solution:

$$t = -\frac{2v_{0y}}{a_y} = -\frac{2v_0 \sin\theta}{a_y} = 2.13 \times 10^{-8} \text{ s}$$

The horizontal distance travelled in this time must be L to make sure the electron strikes the lower plate. Hence,

$$L = v_{0x}t = (v_0 \cos\theta)t = 7.06 \times 10^{-2} \text{ m}$$