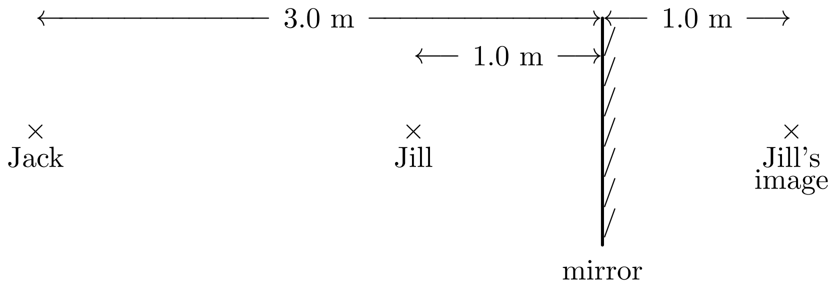
Solutions
Chapter 13

Problem 1



As seen in the figure, the distance between Jack and Jill's image is 4.0m.

Problem 2

The mirror must be concave as a convex mirror cannot give a magnification greater than 1. Also, for convenience in shaving, it would be good to have an upright image! Hence, the magnification would be positive and

$$-i/p = 2.00$$

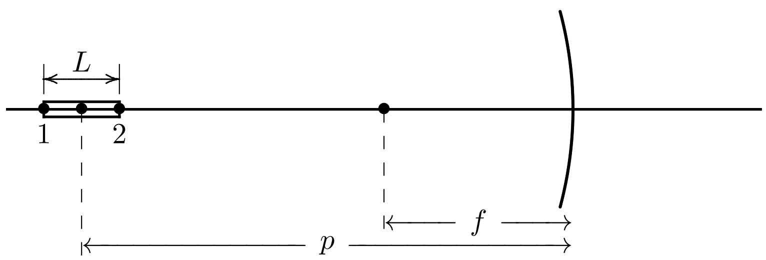
So $i = -2.00p$. The focal length is $f = R/2 = 40.0/2 = 20.0$cm. Then the *imaging formula* gives

$$\frac{1}{-2.00p} + \frac{1}{p} = \frac{1}{20.0}$$

Then

$$p = 10.0 \text{ cm}.$$

Problem 3



Part a

Let p_1 be the distance of end 1 of the object and p_2 the distance of end 2 of the object (from the mirror). So,

$$p_1 = p + L/2$$
$$p_2 = p - L/2$$

The corresponding image positions are (using $1/p + 1/i = 1/f$)

$$i_1 = \frac{fp_1}{p_1 - f}$$
$$i_2 = \frac{fp_2}{p_2 - f}$$

So the image length is

$$\begin{aligned}
L' &= i_1 - i_2 = \frac{fp_1}{p_1 - f} - \frac{fp_2}{p_2 - f} \\
&= \frac{f(p + L/2)}{p - f + L/2} - \frac{f(p - L/2)}{p - f - L/2} \\
&= \frac{f[p^2 - pf - pL/2 + pL/2 - Lf/2 - L^2/4 - p^2 + pf - pL/2 + pL/2 - Lf/2 + L^2/4]}{(p-f)^2 - L^2/4} \\
&= \frac{-Lf^2}{(p-f)^2 - L^2/4} \\
&\simeq -L\left(\frac{f}{p-f}\right)^2
\end{aligned}$$

The last step is an approximation where $L^2/4$ is neglected because L is small. Then

$$m' = \frac{L'}{L} = -\left(\frac{f}{p-f}\right)^2.$$

Part b

The lateral magnification is

$$m = -\frac{i}{p}$$

But (using $1/p + 1/i = 1/f$)

$$i = \frac{fp}{p-f}$$

So

$$m = -\frac{f}{p-f}$$

Hence, the longitudinal magnification is

$$m' = -\left(\frac{f}{p-f}\right)^2 = -m^2$$

Problem 4

Part a

The focal length must be negative as the lens is diverging in nature. Hence, $f = -15$. Using the *imaging formula*:

$$i = \frac{pf}{p-f} = \frac{10 \times (-15)}{10 - (-15)} = -6.0 \text{ cm}.$$

Part b

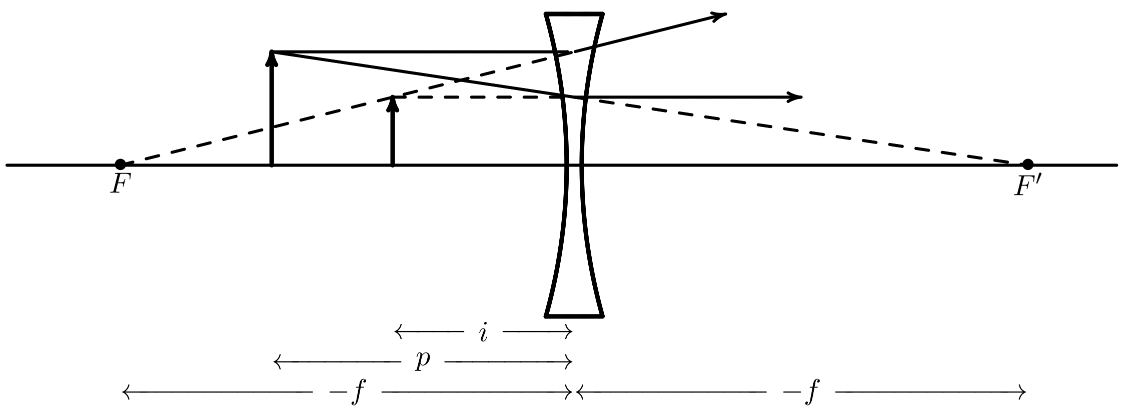
Magnification is

$$m = -i/p = 0.60$$

Part c

The image is virtual as seen in the following ray diagram.

Part d



Problem 5

As p is so much larger than f in this case, $p - f \simeq p$. Hence, the *imaging formula* gives:

$$i = \frac{pf}{p-f} = \frac{pf}{p} = f$$

So, from the definition of magnification, the ratio of the image size to object size is:

$$m = \frac{h'}{h} = \frac{-f}{p}$$

This gives the diameter of the image to be:

$$|h'| = \frac{fh}{p} = \frac{15.0 \times 1.39 \times 10^{11}}{1.50 \times 10^{13}} = 0.139 \text{ cm.}$$

Problem 6

The sum of i and p is 250cm.

$$i + p = 250$$

Also,

$$\frac{1}{i} + \frac{1}{p} = \frac{1}{f} = \frac{1}{10.0}$$

Hence,

$$\frac{1}{250 - p} + \frac{1}{p} = \frac{1}{10.0}$$

$$\frac{(250 - p)p}{250} = 10.0$$

$$p^2 - 250p + 2500 = 0$$

$$p = 240 \text{ cm. or } 10.4 \text{ cm}$$

Hence, the lens could be placed either at 240cm or at 10.4cm to produce a sharp image. However, in most situations one requires an enlarged image from a slide projector. This will be achieved only if $p = 10.4$cm as magnification is given by

$$m = \frac{-i}{p} = -\frac{250 - p}{p}$$

Problem 7

Part a

Only a converging lens can form a real image. So f is positive.

Part b

$$m = \frac{-i}{p} = -2$$

So,

$$i = 2p$$

Also,

$$i + p = 50.0$$

So,

$$2p + p = 50.0$$

$$p = 50.0/3 = 16.7 \text{ cm.}$$

Part c

$$i = 50.0 - 16.7 = 33.3 \text{ cm}$$

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}$$

$$f = \frac{pi}{p+i} = \frac{16.7 \times 33.3}{16.7 + 33.3} = 11.1 \text{ cm.}$$