Solutions
Chapter 11

Problem 1

Part a

$$I = \mathcal{E}_m/Z,$$
$$Z = \sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}$$

As there is no resistor, $R = 0$. As there is no capacitor, $C = \infty$ or $\frac{1}{\omega_d C} = 0$. So $Z = \omega_d L$, and

$$I = \frac{\mathcal{E}_m}{\omega_d L} = 4.17 \times 10^{-3} \text{ A}.$$

Part b

$$i = I \sin(\omega_d t - \phi)$$

where ϕ is given by:

$$\tan\phi = \frac{\omega_d L - \frac{1}{\omega_d C}}{R} = \infty.$$

So, $\phi = \pi/2$ and

$$i = I\sin(\omega_d t - \pi/2) = -I\cos(\omega_d t)$$

Hence, i is a maximum when $\omega_d t = \pi, 3\pi, 5\pi, \ldots$. At these times $\sin(\omega_d t) = 0$. So, at these times, $\mathcal{E} = \mathcal{E}_m \sin(\omega_d t) = 0$.

Problem 2

Part a

$$I = \mathcal{E}_m/Z,$$
$$Z = \sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}$$

As there is no resistor, $R = 0$. As there is no inductor, $L = 0$. So $Z = 1/(\omega_d C)$, and

$$I = \mathcal{E}_m \omega_d C = 5.00 \times 10^{-2} \text{ A}.$$

Part b

$$i = I\sin(\omega_d t - \phi)$$

where ϕ is given by:

$$\tan\phi = \frac{\omega_d L - \frac{1}{\omega_d C}}{R} = -\infty.$$

So, $\phi = -\pi/2$ and

$$i = I\sin(\omega_d t + \pi/2) = I\cos(\omega_d t)$$

Hence, i is a maximum when $\omega_d t = 0, 2\pi, 4\pi, \ldots$. At these times $\sin(\omega_d t) = 0$. So, at these times, $\mathcal{E} = \mathcal{E}_m \sin(\omega_d t) = 0$.

Problem 3

Part a

The resonant frequency is

$$\omega_d = 1/\sqrt{LC} = 500\text{rad/sec}$$

Part b

The current amplitude is

$$I = \frac{\mathcal{E}_m}{Z} = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

At resonance, $X_L = X_C$, hence,

$$I = \frac{\mathcal{E}_m}{R} = \frac{10}{4} = 2.50 \text{ A}$$

Also at resonance

$$\omega_d = \frac{1}{\sqrt{LC}}$$

and the voltage amplitude across the inductor is

$$V_L = IX_L = I\omega_d L = \frac{IL}{\sqrt{LC}} = I\sqrt{\frac{L}{C}} = 2500 \text{ V}$$

Part c

The inductor may have a higher voltage across it than the source because the capacitor voltage can partially (or completely in case of resonance) cancel the inductor voltage.

Problem 4

$$\tan\phi = \frac{X_L - X_C}{R}$$

Hence,

$$R = \frac{X_L - X_C}{\tan\phi} = \frac{\omega_d L - \frac{1}{\omega_d C}}{\tan\phi} = \frac{2\pi f_d L - \frac{1}{2\pi f_d C}}{\tan\phi} = 140\ \ \Omega$$

Problem 5

Part a

$$Z = \sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2} = 67\ \ \Omega$$

Part b

$$I = \frac{\mathcal{E}_m}{Z} = 6.7\ \text{A}$$

Part c

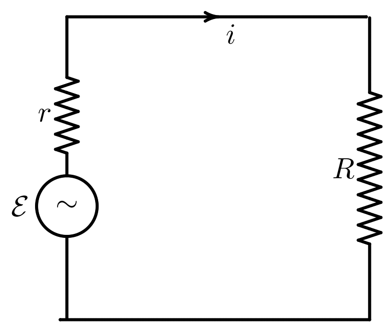
The phase constant is given by

$$\tan\phi = \frac{\omega_d L - 1/(\omega_d C)}{R} = 1.35$$

Hence,

$$\phi = 53^\circ$$

Problem 6



Power dissipation in R is

$$P = I_{\text{rms}}^2 R$$

where

$$I_{\text{rms}} = \frac{\mathcal{E}_{\text{rms}}}{Z} = \frac{\mathcal{E}_{\text{rms}}}{r + R}$$

So,

$$P = \mathcal{E}_{\text{rms}}^2 \frac{R}{(r+R)^2}$$

The value of R for which P is maximum, must obey the maxima condition from calculus: $\frac{dP}{dR} = 0$. This gives:

$$\mathcal{E}_{\text{rms}}^2 \left[\frac{1}{(r+R)^2} - \frac{2R}{(r+R)^3} \right] = 0.$$

The solution to this equation is:

$$r = R.$$

Problem 7

The voltage of the secondary is

$$V_s = N_s \frac{V_p}{N_p} = 4.0 \text{ V}$$