Solutions

Chapter 1

Problem 1

The force is

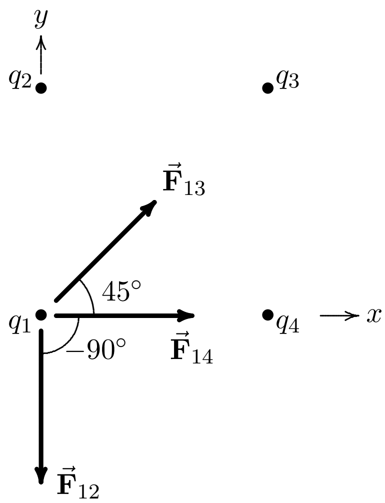
$$F = \frac{k|q_1||q_2|}{r^2}$$

Hence,

$$r = \sqrt{\frac{k|q_1||q_2|}{F}} = \sqrt{\frac{8.99 \times 10^9 \times 25 \times 10^{-6} \times 50 \times 10^{-6}}{8.0}} = 1.19 \text{ m}$$

Problem 2

In the following figure the charges are named as $q_1 = +2q$, $q_2 = +q$, $q_3 = -q$ and $q_4 = -2q$. The x axis is chosen as the left to right direction and the y axis as the bottom to top direction.



The force on charge q_1 due to q_2 is $\vec{\mathbf{F}}_{12}$. From Coulomb's law, the magnitude of this force is,

$$F_{12} = \frac{k|q_1||q_2|}{r_{12}^2} = \frac{2kq^2}{a^2},$$

as a is the distance between the charges. As both charges are positive they repel. So, the force must push q_1 away from q_2 making $\vec{\mathbf{F}}_{12}$ point downward. Hence, this force makes an angle of $-90°$ to the x direction. That means its x and y components are,

$$F_{12x} = F_{12}\cos(-90) = 0, \text{ and } F_{12y} = F_{12}\sin(-90) = -F_{12} = -\frac{2kq^2}{a^2}.$$

Using the notation $\hat{\mathbf{i}}$ for the unit vector in the x direction and $\hat{\mathbf{j}}$ for the unit vector in the y direction, the unit vector form of $\vec{\mathbf{F}}_{12}$ is,

$$\vec{\mathbf{F}}_{12} = F_{12x}\hat{\mathbf{i}} + F_{12y}\hat{\mathbf{j}} = -\frac{2kq^2}{a^2}\hat{\mathbf{j}}.$$

The magnitude of the force between q_1 and q_3 is,

$$F_{13} = \frac{k|q_1||q_3|}{r_{13}^2} = \frac{2kq^2}{2a^2} = \frac{kq^2}{a^2},$$

as r_{13} is the distance between q_1 and q_3 which is the diagonal of the square and, using the Pythagorean theorem, $r_{13}^2 = a^2 + a^2 = 2a^2$. As the two charges have opposite sign, they attract. Hence, the force on q_1 is towards q_3. So, $\vec{\mathbf{F}}_{13}$ is along the diagonal of the square making an angle of 45° to the x axis. This gives,

$$F_{13x} = F_{13}\cos(45) = \frac{kq^2}{a^2}\cos(45) \text{ and } F_{13y} = F_{13}\sin(45) = \frac{kq^2}{a^2}\sin(45).$$

Hence, in unit vector notation,

$$\vec{\mathbf{F}}_{13} = F_{13x}\hat{\mathbf{i}} + F_{13y}\hat{\mathbf{j}} = \frac{kq^2}{a^2}\cos(45)\hat{\mathbf{i}} + \frac{kq^2}{a^2}\sin(45)\hat{\mathbf{j}}$$

The magnitude of the force between q_1 and q_4 is,

$$F_{14} = \frac{k|q_1||q_4|}{r_{14}^2} = \frac{4kq^2}{a^2},$$

as the distance between these charges is $r_{14} = a$. The two charges have opposite sign and hence, they attract. So, charge q_1 is pulled towards q_4 making the force point in the positive x direction. This makes the angle of the force from the x axis to be zero degrees. Hence,

$$F_{14x} = F_{14}\cos(0) = F_{14} = \frac{4kq^2}{a^2} \text{ and } F_{14y} = F_{14}\sin(0) = 0.$$

Then,

$$\vec{\mathbf{F}}_{14} = F_{14x}\hat{\mathbf{i}} + F_{14y}\hat{\mathbf{j}} = \frac{4kq^2}{a^2}\hat{\mathbf{i}}.$$

From the superposition principle, the net force on charge q_1 is,

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{13} + \vec{\mathbf{F}}_{14} =$$

$$= -\frac{2kq^2}{a^2}\hat{\mathbf{j}} + \frac{kq^2}{a^2}\cos(45)\hat{\mathbf{i}} + \frac{kq^2}{a^2}\sin(45)\hat{\mathbf{j}} + \frac{4kq^2}{a^2}\hat{\mathbf{i}}$$

Then, collecting all the $\hat{\mathbf{i}}$ terms and the $\hat{\mathbf{j}}$ terms and substituting the values given for k, q and a,

$$\vec{\mathbf{F}}_1 = \frac{kq^2}{a^2}(\cos(45) + 4)\hat{\mathbf{i}} + \frac{kq^2}{a^2}(-2 + \sin(45))\hat{\mathbf{j}} = 106\hat{\mathbf{i}} - 29\hat{\mathbf{j}} \text{ N}$$

Problem 3

As the net force on q_3 is zero

$$0 = \frac{k|q_1||q_3|}{(3d)^2} - \frac{k|q_2||q_3|}{d^2}$$

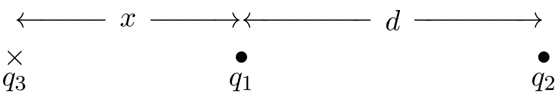
Hence,

$$|q_1|/9 = |q_2|$$

For the forces to be opposite, q_1 and q_2 must be of opposite sign. Hence,

$$q_1 = -9q_2$$

Problem 4



The third charge, q_3, could not be in between q_1 and q_2 as that would cause both forces to be in the same direction and hence they would not cancel to give zero. So q_3 must be either to the left of q_1 or to the right of q_2. As q_2 is larger in magnitude, q_3 must be farther from q_2 than q_1 to allow cancellation of the two forces. So, q_3 must be to the left of q_1 (see figure). The equilibrium condition is

$$\frac{k|q_1||q_3|}{x^2} = \frac{k|q_2||q_3|}{(x+d)^2}$$

Hence,

$$\frac{\sqrt{|q_1|}}{x} = \frac{\sqrt{|q_2|}}{x+d}$$

Solving for x gives

$$x = \frac{\sqrt{|q_1|}d}{\sqrt{|q_2|} - \sqrt{|q_1|}} = 4.8 \text{ m}$$

Problem 5

If e is the charge of a proton, then the electrostatic force magnitude is

$$F_e = \frac{ke^2}{d^2}.$$

If m is the mass of a proton, then the gravitational force magnitude is

$$F_g = \frac{Gm^2}{d^2}.$$

Hence, the ratio of the two forces is

$$\frac{F_e}{F_g} = \frac{ke^2}{Gm^2} = 1.24 \times 10^{36}$$