STATE UNIVERSITY OF NEW YORK
New Paltz, New York.

General Physics 2
First Exam

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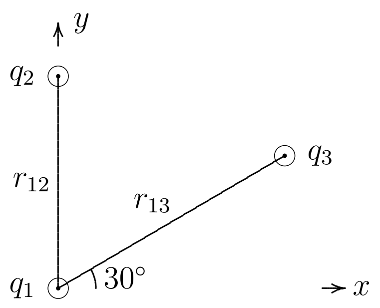
Solutions

Constants and Formulas

$$
\begin{aligned}
k = \frac{1}{4\pi\epsilon_0} &= 9.0 \times 10^9 \quad Nm^2/C^2. \\
F &= k\frac{|q_1||q_2|}{r^2} \\
E &= k\frac{|q|}{r^2} \\
\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} &= q_e/\epsilon_0 \\
V &= k\frac{q}{r}
\end{aligned}
$$

Name________________________________

Problem I



The figure above shows three point charges: $q_1 = 4.0\mu$C, $q_2 = 2.0\mu$C, and $q_3 = 5.0\mu$C. The distances are: $r_{12} = 4.0$m and $r_{13} = 5.0$m. The x and y coordinates are as shown and q_1 is at the origin.

Solution

Question 1

The magnitude of the force is

$$F_{12} = \frac{kq_1q_2}{r_{12}^2} = 4.5 \times 10^{-3}\,\text{N}.$$

The direction is along the negative y direction. Hence,

$$\vec{\mathbf{F}}_{12} = -4.5 \times 10^{-3}\hat{\mathbf{j}}\,\text{N}.$$

Question 2

The magnitude of the force is

$$F_{13} = \frac{kq_1q_3}{r_{13}^2} = 7.2 \times 10^{-3}\,\text{N}.$$

The direction is 210° from the positive x direction. Hence,

$$F_{13x} = F_{13}\cos(210°) = -6.2 \times 10^{-3}\,\text{N}.$$

and

$$F_{13y} = F_{13}\sin(210°) = -3.6 \times 10^{-3}\,\text{N}.$$

Hence,

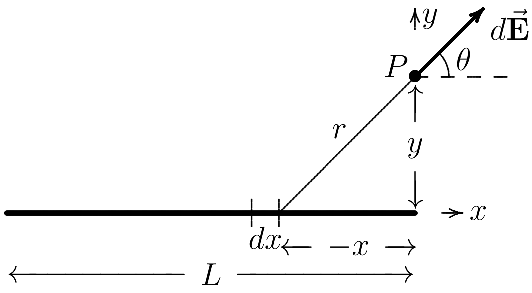
$$\vec{\mathbf{F}}_{13} = -6.2 \times 10^{-3}\hat{\mathbf{i}} - 3.6 \times 10^{-3}\hat{\mathbf{j}}\,\text{N}$$

Question 3

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{13}$$

$$= -6.2 \times 10^{-3}\hat{\mathbf{i}} - 8.1 \times 10^{-3}\hat{\mathbf{j}}\,\text{N}.$$

Problem II



The figure above shows a thin straight wire of length L and uniform charge density λ. The x and y coordinate directions are as shown. The origin is at the right end of the wire. Other symbols to be used are shown in the figure.

Solution

Question 4

The magnitude of the electric field produced by an infnitesimal piece of the wire of length dx is

$$dE = \frac{k\ dq}{r^2} = \frac{k\lambda\ dx}{r^2}.$$

The x component of this is

$$dE_x = dE\cos\theta = \frac{k\lambda\cos\theta\ dx}{r^2}.$$

Question 5

Similarly, the y component is

$$dE_y = dE\sin\theta = \frac{k\lambda\sin\theta\ dx}{r^2}.$$

Question 6

Writing everything in terms of the integration parameter x, we have

$$r = (x^2 + y^2)^{1/2},$$

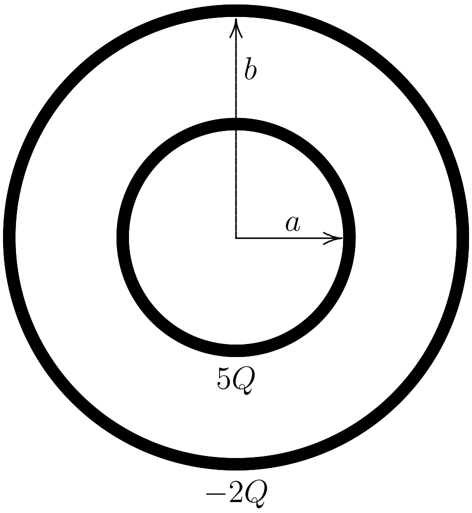
and

$$\cos\theta = -x/r.$$

Hence,

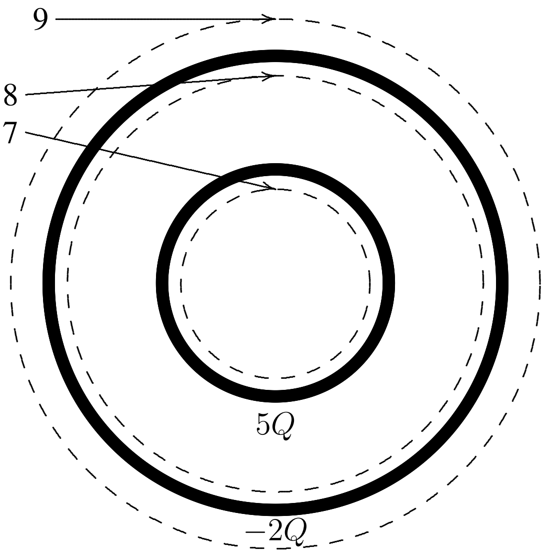
$$E_x = \int dE_x = -k\lambda \int_{-L}^{0} \frac{xdx}{(x^2 + y^2)^{3/2}}.$$

Problem III



The figure above shows two thin conducting concentric spherical shells of radii a and b. The charges placed on the two shells are $5Q$ and $-2Q$ as shown. Q is positive. The system is under static conditions.

Solution



For a spherical Gaussian surface of radius r

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E dA = E \oint dA = E(4\pi r^2)$$

Question 7

For this case, the Gaussian surface is shown in the figure. The enclosed charge for this surface is zero. Hence,

$$E(4\pi r^2) = 0$$

and $E = 0$.

Question 8

For this case, the Gaussian surface is shown in the figure. The enclosed charge for this surface is $5Q$. Hence,

$$E(4\pi r^2) = 5Q/\epsilon_0$$

and $E = 5Q/(4\pi\epsilon_0 r^2)$.

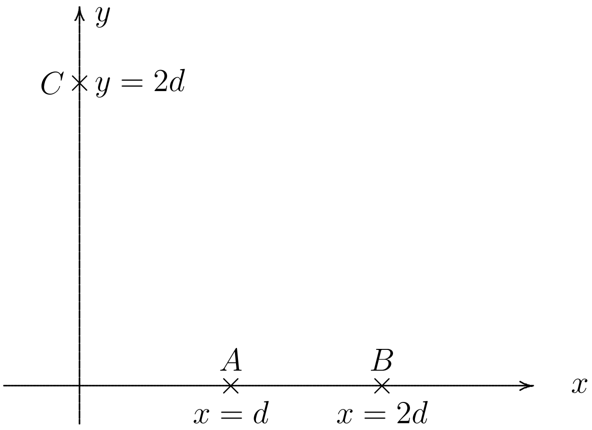
Question 9

For this case, the Gaussian surface is shown in the figure. The enclosed charge for this surface is $3Q$. Hence,

$$E(4\pi r^2) = 3Q/\epsilon_0$$

and $E = 3Q/(4\pi\epsilon_0 r^2)$.

Problem IV



The figure above shows a coordinate system with three positions (A, B and C) marked with a '$\times$'. A and B are on the x axis at $x = d$ and $x = 2d$ respectively. C is on the y axis at $y = 2d$.

Solution

Question 10

$$V = \frac{kQ}{2d} + \frac{k(-Q)}{2d} = 0.$$

Question 11

$$V = \frac{kQ}{d} + \frac{k(-Q)}{2d} = \frac{kQ}{2d}.$$

Question 12

As the two charges each produce non-zero electric fields, one in the x direction and the other in the y direction, they cannot cancel each other. Hence,

$$\vec{\mathbf{E}} \neq 0.$$