Revisiting μ and τ as Excitations of the Electron in Light of CLFV*

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Charged lepton flavor violation (CLFV) is an interesting phenomenon to investigate in going beyond the Standard Model (BSM). This direction of investigation also inspires a new look at the idea of μ and τ being excitations of the electron (e). For this, the electron is required to have a substructure that is held together by some potential. However, even the simplest model of a two-body substructure has several troubling issues. First, a relativistically covariant formulation of such a bound system is non-trivial. However, this has been resolved in the past in a different context. Second, a consistent field theory of composite objects is needed to handle this model of leptons with substructure. This has also been done in the past in a different context. Third, the large observed mass ratios ($m_\mu/m_e = 206.6$) of e and μ and ($m_\tau/m_\mu = 16.81$) of μ and τ rule out binding potentials that depend only on the relative positions of constituents. Here it is shown that a concept similar to the "running coupling constant" of strong interactions generates a model that fits these ratios very well ($m_\mu/m_e = 206.6$, $m_\tau/m_\mu = 18.49$).

I. INTRODUCTION

Efforts at going beyond the Standard Model (BSM) are picking up rapidly in recent years. In particular, the search for charged lepton flavor violation (CLFV) is an interesting direction[1, 2]. Historically, the discovery of μ was followed by the consideration of it being an excitation of the electron (e)[3]. Hence, the radiative decay of μ to e with photon emission ($\mu \to e\gamma$) was considered natural. However, the lack of observation of such decay led to the selection rule against flavor violation. Present searches of flavor violation do not necessarily indicate that μ and τ are excitations of e. But it is a possibility. It is also possible that the three charged leptons are structureless and they transition from one to another through field interactions. This paper considers the first possibility – the possibility of these particles being different eigenstates of the same bound composite object. For this purpose, the usual quantum field theory (QFT) of structureless electrons needs to be extended to a composite particle quantum field theory (CQFT)[4–6]. In such a model, the composite electron mass turns out to be the energy eigenvalue of a variation of the time-independent Klein-Gordon equation[5].

II. ELECTRON AS A COMPOSITE

For simplicity, let us consider the electron to be composed of a spin-half fermionic vertex particle and a spin-zero bosonic satellite[6–8] held together by a scalar potential introduced as a mass function of the satellite. In the center of mass (CM) frame, the vertex can be assigned zero energy. This makes the energy E of the satellite alone the total energy of the composite in the CM frame which is, by definition, its mass[9][12]. Such an assignment can be made without loss of generality due to the composite nature of the electron. Hence, the ground state value of E is the electron mass and μ and τ masses are excited state values of E. The full equation of state for the composite can be decoupled such that the satellite part has the appearance of a Klein-Gordon equation as follows[6].

$$(\partial_\mu \partial^\mu - m_s^2)\psi = 0, \tag{1}$$

where ∂_μ is the relativistic gradient operator, ψ the wave-function and m_s a scalar function of r the radial coordinate of the satellite position relative to the vertex in the CM frame. Unlike in the free particle Klein-Gordon equation, m_s is not a constant. We shall call it the "mass function". Here, it provides the binding for the composite. The time-independent version of equation 1 in the CM frame provides the CM energy E of the satellite which is also the mass of the composite. It is as follows.

$$\nabla^2\Psi - m_s^2\Psi + E^2\Psi = 0, \tag{2}$$

where Ψ is the time-independent part of ψ. As m_s has to be spherically symmetric in the CM frame, the angular part of Ψ is given by the usual spherical harmonics Y_l^m. Hence,

$$\Psi = R_{nl}(r)Y_l^m(\theta, \phi), \tag{3}$$

where, (r, θ, ϕ) are the usual polar coordinates in the CM frame and R_{nl} is the radial part of the wave-function. Suppressing the subscripts of R_{nl}, the radial equation is as follows.

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) - \frac{l(l+1)}{r^2}R + (E^2 - m_s^2)R = 0, \tag{4}$$

where l is the usual total angular momentum quantum number.

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III. A "RUNNING COUPLING CONSTANT" DEPENDING ON MASS

The mass function m_s can provide a binding effect for the satellite if it is an appropriate function of r. So, it plays the role of a binding scalar potential. However, no ordinary function of r seems to produce anything remotely close to the very large observed mass ratios[7] m_μ/m_e and m_τ/m_μ where m_e, m_μ and m_τ are the observed masses of e, μ and τ.

$$m_\mu/m_e = 206.6, \quad m_\tau/m_\mu = 16.81. \tag{5}$$

Hence, it is evident that some unconventional choices need to be made. One such choice is to have m_s include the energy E in its functional form. Note that E, the energy eigenvalue in equation 4, is the mass of the composite, and three of its values are the masses m_e, m_μ and m_τ. Of course, this makes equation 4 depend on E in a nontrivial way. Nonetheless, it has physically meaningful solutions for appropriately chosen mass functions. In this paper, the following function is seen to provide surprising agreement with experiment.

$$m_s^2 = \Lambda E(Er)^q, \tag{6}$$

where Λ is a fitted parameter with the dimension of mass and q is a dimensionless positive power. As we are working with natural units where $\hbar = c = 1$, (Er) can be seen as a dimensionless radial coordinate. Hence, we may write,

$$m_s^2 = \Lambda E(\rho)^q, \tag{7}$$

where,

$$\rho = Er. \tag{8}$$

One may consider Λ to be the analog of a usual coupling constant. However, from the point of view of the usual radial coordinate r, Λ is not the coupling constant by itself. The dependence of m_s on E adds to the effect of a coupling constant. This is akin to the "running coupling constant" of strong interactions.

Dividing equation 4 by E^2 produces the following.

$$\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{dR}{d\rho}\right) - \frac{l(l+1)}{\rho^2}R + \left(1 - \frac{\rho^q}{\epsilon}\right)R = 0, \tag{9}$$

where,

$$\epsilon = \frac{E}{\Lambda}. \tag{10}$$

Equation 9 can be solved numerically to find the energy eigenvalues E.

IV. THE NUMERICAL METHOD

Equation 9 is prepared for a numerical solution by the following substitution.

$$R(\rho) = \rho^s G(\rho). \tag{11}$$

It gives

$$\frac{d^2G}{d\rho^2} + \frac{2(s+1)}{\rho}\frac{dG}{d\rho} - \frac{l(l+1) - s(s+1)}{\rho^2}G + \left(1 - \frac{\rho^q}{\epsilon}\right)G = 0. \tag{12}$$

For solutions that are non-singular at the origin, we must choose,

$$s = l, \quad G'(0) = 0 \quad \text{and} \quad G(0) \neq 0. \tag{13}$$

This gives,

$$\frac{d^2G}{d\rho^2} + \frac{2(l+1)}{\rho}\frac{dG}{d\rho} + \left(1 - \frac{\rho^q}{\epsilon}\right)G = 0. \tag{14}$$

To use the Runge-Kutta method, we split the above second order equation into two first order equations by the following substitutions.

$$y_0 = G, \quad y_1 = G'. \tag{15}$$

The resulting first order equations are,

$$\frac{dy_0}{d\rho} = y_1, \tag{16}$$

$$\frac{dy_1}{d\rho} = \left(\frac{\rho^q}{\epsilon} - 1\right)y_0 - \frac{2(l+1)}{\rho}y_1, \quad \text{for } \rho \neq 0. \tag{17}$$

For the $\rho = 0$ case of equation 17, the following first-order approximation for y_1 is used on the right-hand-side for $\rho \to 0$.

$$y_1(\rho) = y_1(0) + \rho y_1'(0). \tag{18}$$

As $y_1(0) = G'(0) = 0$ (from the initial value equations 13) and q is positive, this gives,

$$\frac{dy_1}{d\rho} = -\frac{y_0(0)}{1 + 2(l+1)}, \quad \text{for } \rho = 0. \tag{19}$$

With this form of the equations, the shooting method is used to numerically search for the eigenvalues ϵ.

V. RESULTS

In the following, the eigenvalues ϵ are labeled by the subscripts n and l. n is the number of nodes of the function R_{nl} including the node at infinity and l is the usual total angular momentum quantum number. Note that here n is not the usual hydrogen atom quantum number. It does not include l in its definition. Hence, for each n the possible values of l are not limited. Also, the corresponding values of E are given by equation 10 to be,

$$E_{nl} = \Lambda\epsilon_{nl}. \tag{20}$$

Note that the mass ratios are independent of the fitted parameter Λ. Then, using the half-integer value,

$$q = 13/2, \tag{21}$$

for the only other adjusted parameter, the theoretical mass ratios are computed to be,

$$\frac{m_\mu}{m_e} = \frac{E_{20}}{E_{10}} = 206.6, \quad \frac{m_\tau}{m_\mu} = \frac{E_{30}}{E_{20}} = 18.49. \tag{22}$$

This shows strong agreement with the experimental values of equation 5. It is interesting to compute the following factor from the Koide formula[10] using these values.

$$Q = \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = 0.6762. \tag{23}$$

This factor of Q is also independent of Λ and its observed value is $2/3$ to five significant figures.

To fit the actual masses, we need,

$$\Lambda = 8.9687 \times 10^{-4} \text{ Mev}. \tag{24}$$

Then the fitted values for the lepton masses are,

$$m_e = E_{10} = 0.5117 \text{ Mev}, \tag{25}$$
$$m_\mu = E_{20} = 105.7 \text{ Mev}, \tag{26}$$
$$m_\tau = E_{30} = 1955 \text{ Mev}. \tag{27}$$

Here it is assumed that non-zero values for l are difficult to observe in practice. This is similar to the excited states of mesons.

The observed values for the masses are,

$$m_e = 0.5117\text{Mev}, \quad m_\mu = 105.7 \text{ Mev}, \quad m_\tau = 1777 \text{ Mev}. \tag{28}$$

VI. CONCLUSION

Here, the three charged leptons are considered to be different energy states of the same composite object[11]. However, ordinary binding potentials are unable to produce the large differences in the masses m_e, m_μ and m_τ. So, a mass function that depends both on the separation of the constituents and the total energy is used. This allows the fitted values to be very close to the observed ones.

The results presented here are preliminary. Further consideration must be given to the following issues besides others.

- Other mass functions might produce better fits to experiment.
- More elaborate models are possible using a spin-one satellite.

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